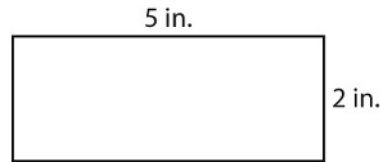


LESSON
8-1

Similar Shapes and Scale Drawings

Practice and Problem Solving: A/B

1. The plan of a terrace is shown at right. The scale is 2 inches : 6 feet. What are the length and width of the terrace? Find the terrace's area.

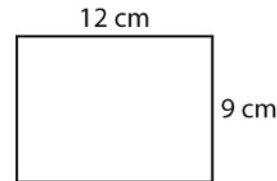


Length: _____

Width: _____

Area: _____

2. The floor plan of a ballroom is shown at right. The scale is 3 centimeters : 4 meters. What are the length and width of the ballroom? Find the ballroom's area.



Length: _____

Width: _____

Area: _____

3. A garage floor measures 150 feet by 120 feet. A scale drawing of the floor on grid paper uses a scale of 1 unit : 15 feet. What are the dimensions of the drawing?

4. The scale model of a skyscraper being built is 4.2 feet tall.
a. When it is finished, the skyscraper will be 525 meters tall. What scale was used to make the model?

- b. The model is made out of a stack of plywood sheets. Each sheet is 0.6 inch thick. How many sheets of plywood tall is the model?

5. You have been asked to build a scale model of a restaurant out of bottle caps. The restaurant is 20 feet tall. Your scale is 2.4 cm : 1 foot.
a. A bottle cap is 1.2 cm tall. About how many bottle caps tall will your model be?

- b. You are out of bottle caps, and decide to use popsicle sticks instead. You measure them, and they are 15.2 cm tall. How many popsicle sticks tall will your model be?

LESSON
8-1

Similar Shapes and Scale Drawings

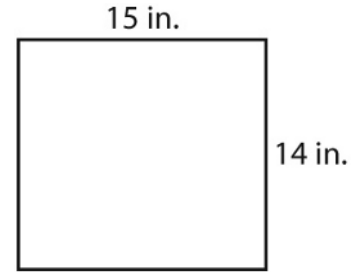
Practice and Problem Solving: C

1. A scale drawing of a patio is shown at right. The scale is 4 inches : 6.8 feet. What are the length and width of the patio? Find the patio's area.

Length: _____

Width: _____

Area: _____



2. The scale for a scale drawing is 8 millimeters : 1 centimeter. Which is larger, the actual object or the scale drawing? Explain.

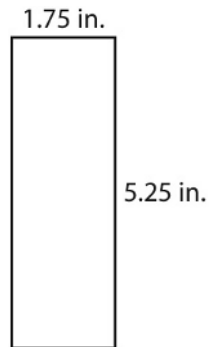
3. Carol has a small copy of a painting. The dimensions of her copy are shown at right. The scale of the copy is 0.5 inches : 12 centimeter.

- a. Find the dimensions of the original painting.

- b. Find the area of the original painting.

- c. One centimeter is equal to approximately 0.033 foot. Find the dimensions of the original painting in feet. Round to the nearest thousandth.

- d. Find the area of the original painting in square feet. Round to the nearest thousandth.



4. On a scale drawing, a bookshelf is 8 inches tall. The scale factor is $\frac{1}{8}$.

What is the height of the bookshelf?

5. On a scale drawing, a school is 1.6 feet tall. The scale factor is $\frac{1}{22}$.

Find the height of the school.

LESSON
8-1

Similar Shapes and Scale Drawings

Practice and Problem Solving: D

1. A room in a house is shown on a blueprint. The blueprint has a scale of 5 inches : 8 feet. A wall in the same blueprint is 30 inches. Complete the table. The first column is done for you.

Blueprint Length (in.)	5	10		20		
Actual Length (ft)	8	16	24		40	48

- a. How long is the actual wall? _____
- b. A door in the room has a width of 4 feet. What is the width of the door in the blueprint?

2. The scale of a room in a blueprint is 2 inches : 1 foot. A window in the same blueprint is 12 inches. Complete the table.

Blueprint Length (in.)	2	4		8		12
Actual Length (ft)	1		3		5	

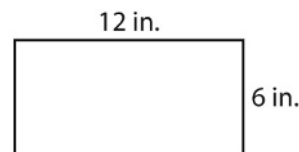
- a. How long is the actual window? _____
- b. A mantel in the room has an actual width of 8 feet. What is the width of the mantel in the blueprint?

3. The scale in the drawing is 2 inches : 4 feet. What are the length and width of the actual room? Find the area of the actual room.

Length: _____

Width: _____

Area: _____



4. A studio apartment has a floor that measures 80 feet by 64 feet. A scale drawing of the floor on grid paper uses a scale of 1 unit : 8 feet. What are the dimensions of the scale drawing?

LESSON
8-1

Similar Shapes and Scale Drawings

Reteach

The dimensions of a scale model or scale drawing are related to the actual dimensions by a *scale factor*. The **scale factor** is a ratio.

The length of a model car is 9 in. \longrightarrow $\frac{9 \text{ in.}}{162 \text{ in.}} = \frac{9 \div 9}{162 \div 9} = \frac{1}{18}$
 The length of the actual car is 162 in. \longrightarrow

$\frac{9}{162}$ can be simplified to $\frac{1}{18}$. The scale factor is $\frac{1}{18}$.

If you know the scale factor, you can use a proportion to find the dimensions of an actual object or of a scale model or drawing.

- The scale factor of a model train set is $\frac{1}{87}$. A piece of track in the model train set is 8 in. long. What is the actual length of the track?
 $\frac{\text{model length}}{\text{actual length}} = \frac{8}{x} \quad \frac{8}{x} = \frac{1}{87} \quad x = 696$

The actual length of track is 696 inches.

- The distance between 2 cities on a map is 4.5 centimeters. The map scale is 1 cm : 40 mi.
 $\frac{\text{distance on map}}{\text{actual distance}} = \frac{4.5 \text{ cm}}{x \text{ mi}} = \frac{1 \text{ cm}}{40 \text{ mi}}$

$$x = 180$$

The actual distance is 180 miles.

Identify the scale factor.

1. Photograph: height 3 in.
 Painting: height 24 in.

$$\frac{\text{photo height}}{\text{painting height}} = \frac{\text{in.}}{\text{in.}} = \underline{\hspace{2cm}}$$

2. Butterfly: wingspan 20 cm
 Silk butterfly: wingspan 4 cm

$$\frac{\text{silk butterfly}}{\text{butterfly}} = \frac{\text{cm}}{\text{cm}} = \underline{\hspace{2cm}}$$

Solve.

3. On a scale drawing, the scale factor is $\frac{1}{12}$. A plum tree is 7 inches tall on the scale drawing. What is the actual height of the tree?

4. On a road map, the distance between 2 cities is 2.5 inches. The map scale is 1 inch:30 miles. What is the actual distance between the cities?

LESSON
8-1

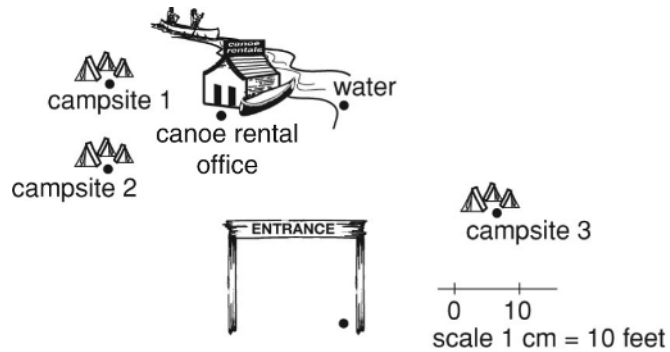
Similar Shapes and Scale Drawings

Reading Strategies: Read a Map

A **scale drawing** has the same shape, but is not the same size, as the object it represents. A map is an example of a scale drawing.

This is a map of a campground. The scale is 1 cm:10 ft.

To find how far the campground entrance is from the canoe rental office, follow the steps. Use a centimeter ruler to measure.



Step 1: Measure the distance in centimeters.

→ The distance is 3 centimeters.

Step 2: Set up a proportion using the map scale as one ratio.

→
$$\frac{1 \text{ cm}}{10 \text{ ft}} = \frac{3 \text{ cm}}{x \text{ ft}}$$

Step 3: Use the proportion. → $x = 3 \cdot 10$

Step 4: Solve to find the value of x. → $x = 30$

The campground entrance is 30 feet from the canoe rental office.

Use the map to answer each question.

1. How many centimeters is Campsite 3 from the water?

2. Write a proportion to find the distance from Campsite 3 to the water.

3. How many centimeters is Campsite 3 from the canoe rental office?

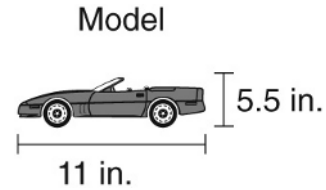
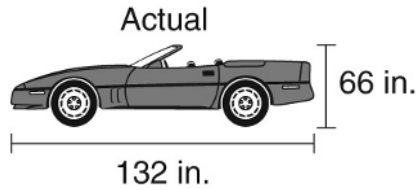
4. Write a proportion to find the distance from Campsite 3 to the canoe rental office.

LESSON
8-1

Similar Shapes and Scale Drawings

Success for English Learners

Problem 1



What is the scale factor?

$$\frac{\text{Model length}}{\text{Actual length}} = \frac{11}{132} = \frac{1}{12}, \quad \frac{\text{Model height}}{\text{Actual height}} = \frac{5.5}{66} = \frac{1}{12}$$

$$\text{Scale factor} = \frac{1}{12}$$

Problem 2

This is a photo of a painting. If you measure the photo, you could find the measurements of the actual painting.



$$\text{Scale factor is } \frac{1}{15}$$

$$\frac{1}{15} \rightarrow \frac{\text{photo}}{\text{painting}}$$

Write 2 proportions, one for the length and one for the width.

$$\frac{\text{photo length}}{\text{painting length}} = \frac{1}{15}$$

$$\frac{\text{photo width}}{\text{painting width}} = \frac{1}{15}$$

1. In Problem 1, what would happen if you used a different scale factor for the length than you did for the width?

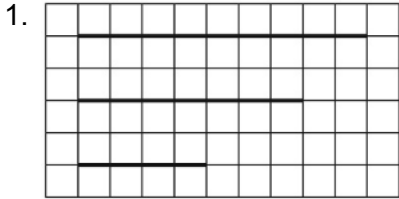
2. Explain why it is important for the photo and the painting to be in proportion in Problem 2.

LESSON
8-2

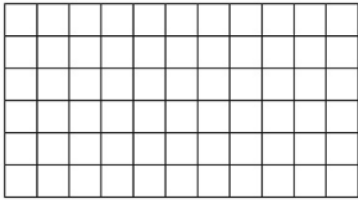
Geometric Drawings

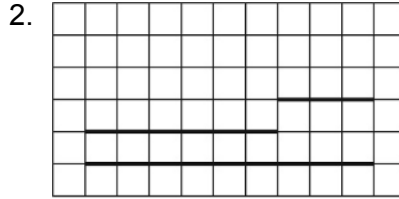
Practice and Problem Solving: A/B

Use each set of line segments to sketch a triangle. If a triangle cannot be drawn, explain why.

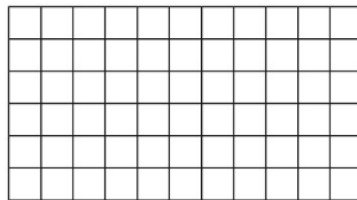


Sketch:





Sketch:



Can each set of line segments form a triangle? Why or why not?

3. $\overline{AB} = \frac{1}{2}$ mile

$\overline{BC} = \frac{1}{3}$ mile

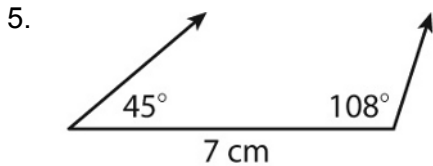
$\overline{AC} = \frac{1}{4}$ mile

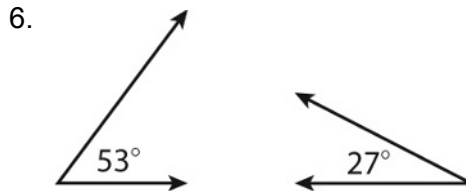
4. $\overline{DE} = 0.205$ kilometer

$\overline{EF} = 0.01$ kilometer

$\overline{DF} = 0.02$ kilometer

How many triangles are formed by the angles and sides—unique triangle, no triangle, or many triangles?



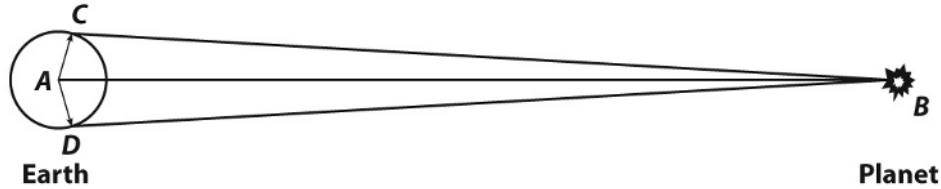


LESSON
8-2

Geometric Drawings

Practice and Problem Solving: C

Astronomers often use triangles to compute the distance between Earth and other planets in the solar system. The diagram below shows how triangles are used in this process.



1. Triangles ABC and ABD are right triangles. Where are the right angles?

2. What is the distance AC in this diagram called? (*Hint: What part of the circle is it?*) How does it compare to AD ?

3. Without using exact numbers, compare the distance AC with the distance BC ?

4. Without using exact numbers, compare the distances AB and BC .

5. Since ABC is a right triangle, how does the distance AB compare to BC ? Is it equal, greater than, or less than? Why?

6. If you did **not** know that ABC is a right triangle, what kind of triangle would Exercise 4 suggest that ABC is? Why?

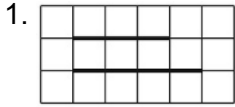
7. The astronomer would like to know the distance AB from Earth's center to the planet. The astronomer knows the distance AC and that ABC is a right triangle. What other angles or sides could the astronomer measure to help find the distance AB ?

LESSON
8-2

Geometric Drawings

Practice and Problem Solving: D

Answer the questions. The first one is started for you.



How long are the two line segments?

3 units and 4 units

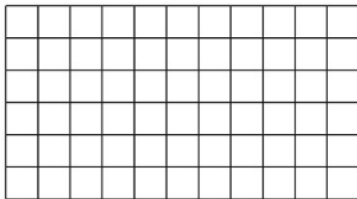
To form a triangle, how long must a third line segment be?

less than 7 units but greater

than 1 unit

Choose a third side length, and then draw the triangle on the grid.

Third side length: _____

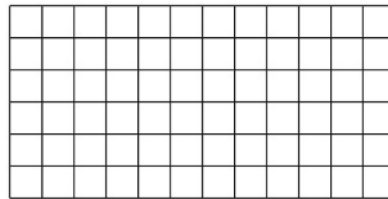


How long are the two line segments?

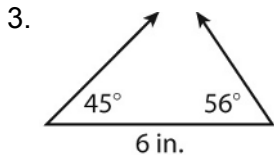
To form a triangle, how long must a third line segment be?

Choose a third side length, and then draw the triangle on the grid.

Third side length: _____



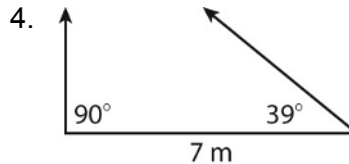
Answer the questions. The first one is started for you.



What is the sum of the two angle measures?

101°

If these angles formed a triangle, what would be the measure of the third angle?



What is the sum of the two angle measures?

If these angles formed a triangle, what would be the measure of the third angle?

LESSON
8-2

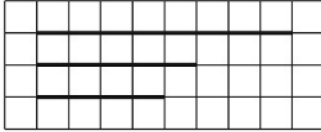
Geometric Drawings

Reteach

In this lesson, you learned two different sets of conditions for drawing a triangle.

Three Sides

Can these three sides form a triangle?



The condition that a triangle can be formed is based on this fact:

The sum of the lengths of two shorter sides is greater than the length of the longest side.

What are the lengths of the shorter sides?

4 and 5 units

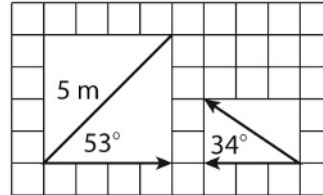
What is the length of the longest side?

8 units

Is $4 + 5 > 8$? Yes.

Two Angles and a Side

Why is a common, or included, side needed? Do these angles and side form a triangle?



The condition that a triangle can be formed is based on this fact:

The sum of the measures of the angles in a plane triangle is 180 degrees.

What would be the measure of the third angle in a triangle formed from these parts?

$$180^\circ = 53^\circ + 34^\circ + x^\circ$$

$$x^\circ = 180^\circ - 87^\circ$$

$$x = 93^\circ$$

A triangle can be formed, with the angles 53° and 93° having the 5-meter side in common.

Answer the questions about triangle drawings.

1. Can a triangle be formed with three sides of equal length? Explain using the model above.

2. Can a triangle be formed with angles having measures of 30° , 70° , and 110° ? Explain using the model above.

LESSON
8-2**Geometric Drawings****Reading Strategies: Using Graphic Aids**

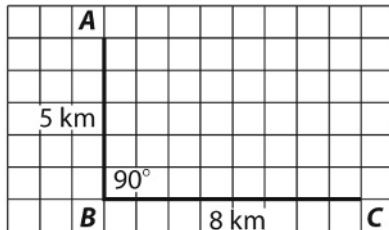
Real-world problems can be solved more easily in some cases when you draw a diagram to represent the situation.

Example

Two roads meet at a right angle. One road is 5 kilometers in length, and the other is 8 kilometers in length. A third road is constructed connecting the ends of the other two roads. What is its minimum length?

Solution

First sketch the situation.



The two sides, AB , and BC , are the legs of a right triangle. A straight line connecting points A and C would form the hypotenuse. It would also be the shortest distance between points A and B .

“Minimum length” and “shortest distance” mean the same thing here.

Since points A , B , and C form a right triangle, the Pythagorean Theorem can be used to find the length of line segment AC .

$$AC^2 = AB^2 + BC^2, \text{ so } AC^2 = 5^2 + 8^2 = 25 + 64 = 89; \sqrt{AC^2} = AC = \sqrt{89}.$$

So, the length of the road represented by AC is $\sqrt{89}$ kilometers.

Check

Use this result to check that three sides form a triangle. The long side, $\sqrt{89}$, is between $\sqrt{81}$, or 9, and $\sqrt{100}$, or 10. Add 5 and 8 to get 13. Since $13 > 9$ and $13 > 10$, the sides form a triangle.

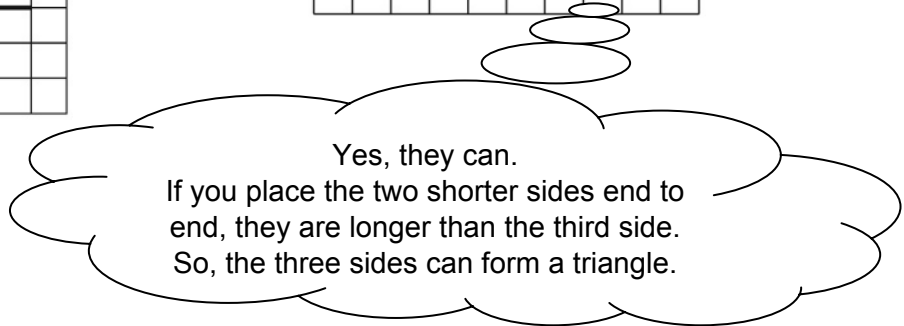
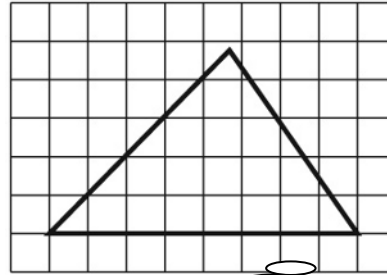
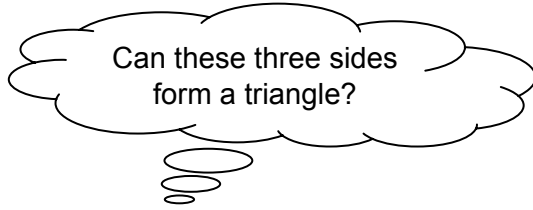
Use another sheet of paper to answer the questions.

1. A carpenter has a 10-foot board and two 4-foot boards. Can these form a triangular wooden brace? Explain.
2. A father is giving his child a 30-inch long softball bat for her birthday. He has a rectangular box that has the dimensions of 5 inches by 6 inches by 25 inches. Will the bat fit in the box? Explain.

LESSON
8-2

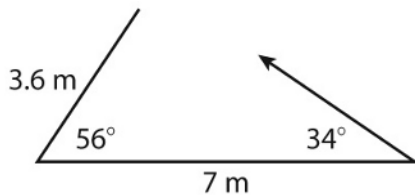
Geometric Drawings
Success for English Learners

Problem 1



Problem 2

Describe the triangle formed by these parts and any others you can compute or measure.



$$180^\circ = 56^\circ + 34^\circ + 90^\circ$$

How long are the sides?

3.6 m, 6 m, and 7 m.

Answer the questions.

1. In Problem 1, how would using a compass and a ruler make drawing the triangle easier?

2. Is the triangle in Problem 2 a right triangle? How do you know?

LESSON
8-3

Cross Sections

Practice and Problem Solving: A/B

What is the common set of points for these figures called—an *intersection* or a *cross section*? Place a check mark by the correct name. Describe the geometric figure formed by the common points. Assume that the two figures have more than one point in common.

1. A circle and the lateral surface of a cone.

Cross section _____ Intersection _____

Figure formed: _____

2. The edge of a square and the base of a pyramid.

Cross section _____ Intersection _____

Figure formed: _____

3. A plane that is perpendicular to the base of a cube and slices through the cube.

Cross section _____ Intersection _____

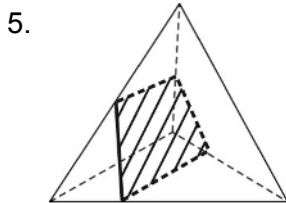
Figure formed: _____

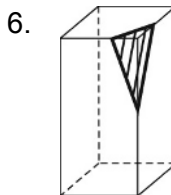
4. A circle with an area bigger than the base of a pyramid and slicing parallel to the base through the pyramid between its apex and its base.

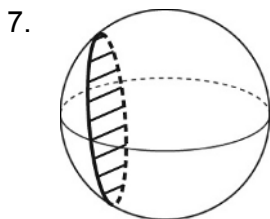
Cross section _____ Intersection _____

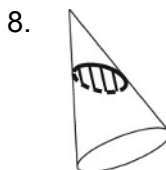
Figure formed: _____

Name or describe the geometric figure that is shaded. Each shaded region results from a plane passing through the solid.









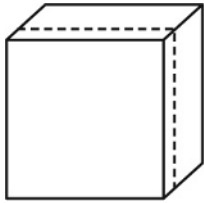
LESSON
8-3

Cross Sections

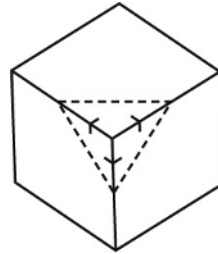
Practice and Problem Solving: C

Two identical cubes are intersected by a plane, resulting in two different cross sections as shown. Answer the questions about the two cross sections.

Cross section X



Cross section Y



1. The plane that created cross section X sliced through the cube parallel to one of its sides. What type of figure is this? Explain.

2. The plane that created cross section Y sliced through the corner of the cube in such a way that it intersected the midpoints of the sides of the cube. What type of triangle is formed by this cross section? Explain.

A plane intersects a cylinder. In Diagram A, the plane is perpendicular to the long axis of the cylinder. In Diagram B, the plane is tilted relative to the cylinder axis. In Diagram C, the plane is tilted even more. In Diagram D, the plane passes through the cylinder along its axis. Answer the questions.

Diagram A

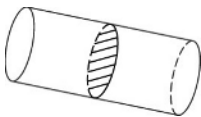


Diagram B

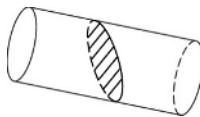


Diagram C

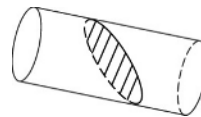
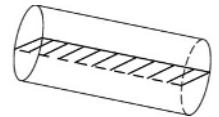


Diagram D



3. What shape are the cross sections in Diagrams A, B, C and D?

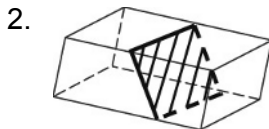
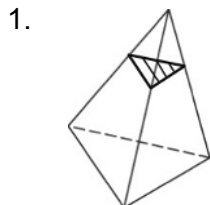
4. Compare the areas of the four cross sections.

LESSON
8-3

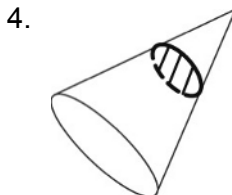
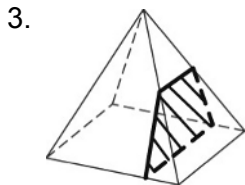
Cross Sections

Practice and Problem Solving: D

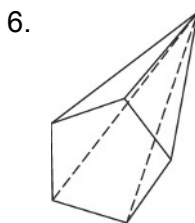
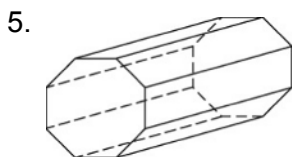
Describe the shape that is formed by the cross section. The first one is done for you.



a triangle that is similar to the base



Draw cross sections for these figures that are parallel to their bases.



LESSON
8-3

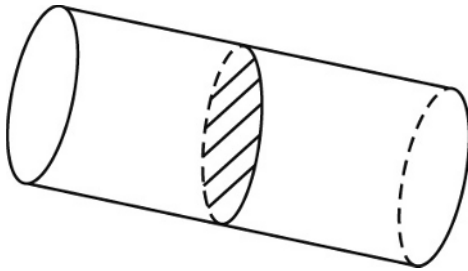
Cross Sections

Reteach

Cross sections can take a variety of shapes, but they are generally related to the parts of the figures from which they are formed. The angle at which the intersecting plane “cuts” the figure is also a factor in determining the shape of the cross section. However, the cross section is always defined as a plane figure in the situations presented here.

Example 1

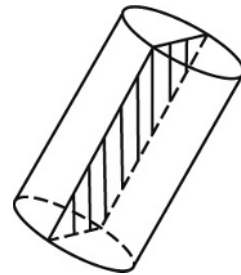
When the intersecting plane is *parallel* to the base(s) of the figure, the cross section is often related to the shape of the base. In this cylinder, the cross section is congruent to the bases.



What is the shape of the cross section?
The cross section is a circle that is congruent to each of the bases of the cylinder.

Example 2

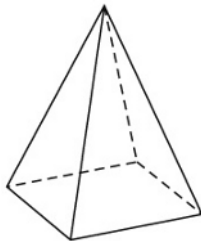
When the intersecting plane is *perpendicular* to the base(s) of the figure, the cross section is not always the same shape as the base. In this cylinder, the cross section is a rectangle, not a circle.



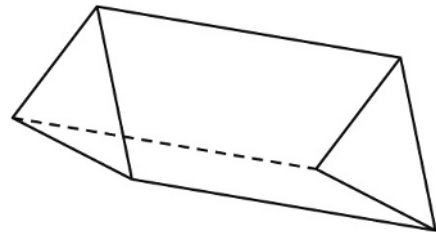
What is the cross section?
A rectangle having a length equal to the height of the cylinder and a width equal to the diameter of the cylinder.

For each solid, draw at least two cross sections with two different shapes. Describe the cross sections.

1.



2.



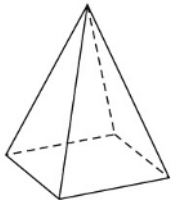
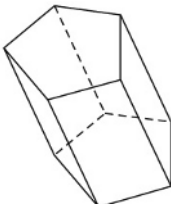
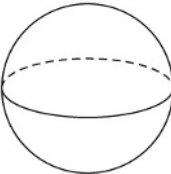

LESSON
8-3

Cross Sections

Reading Strategies: Use a Table

Cross sections are often related to parts of the solids from which they are formed. This makes it possible to organize into a table, information about each solid, its cross section, and the plane that “cut” the figures to form the cross section.

Draw each cross section as described. Complete the table. Be as specific in your descriptions as possible.

Solid	Intersection Direction	Cross Section Shape	How Cross Section Shape Relates to the Solid
1. 	parallel to the rectangular base	2.	similar to base
3. 	4.	a regular pentagon	5.
6. 	passing through the sphere but not through its center	7.	8.
9. 	passing through the cone's vertex and bisecting its base	10.	11.

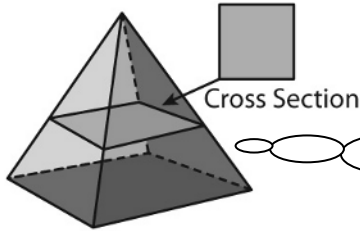
LESSON
8-3

Cross Sections

Success for English Learners

Problem 1

How can you pass a plane through the pyramid so that the cross section is a rectangle?



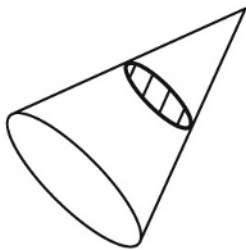
Any plane parallel to the base of the pyramid will make a cross section that is a rectangle.

Problem 2

Cross sections of a cone and a pyramid are shown below.

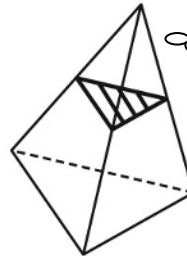
How are the two cross sections different?

Cone



Cross section: circle

Pyramid



Cross section: triangle

Name the **shapes** formed by the cross sections.

Answer the questions.

1. In Problem 1, if the cross section has 4 sides, and one side lies in the base of the pyramid, what shape is the cross section? Explain.

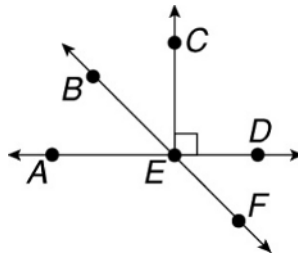
2. In Problem 2, how are the cross sections of the cone and pyramid alike?

LESSON
8-4

Angle Relationships

Practice and Problem Solving: A/B

For Exercises 1–3, use the figure.

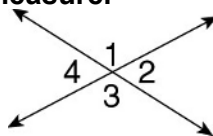


1. Name a pair of vertical angles.

2. Name a pair of complementary angles.

3. Name a pair of supplementary angles.

Use the diagram to find each angle measure.



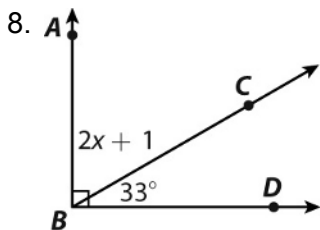
4. If $m\angle 1 = 120^\circ$, find $m\angle 3$.

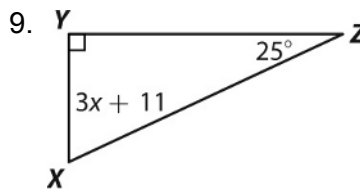
5. If $m\angle 2 = 13^\circ$, find $m\angle 4$.

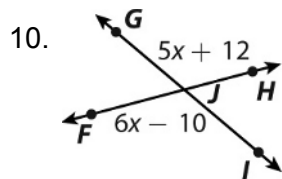
6. If $m\angle 3 = 110^\circ$, find $m\angle 2$.

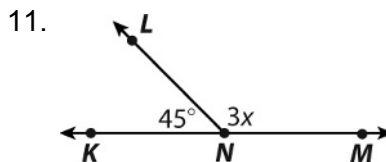
7. If $m\angle 4 = 65^\circ$, find $m\angle 1$.

Find the value of x in each figure.







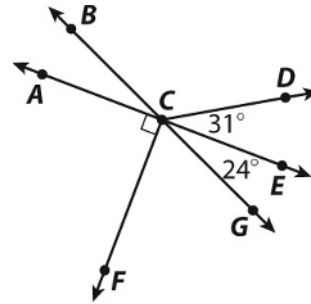


LESSON
8-4

Angle Relationships

Practice and Problem Solving: C

For Exercises 1–4, use the figure.



1. Find $m\angle FCG$.

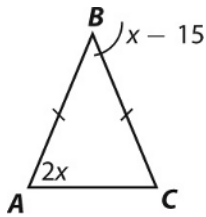
2. Find $m\angle BCD$.

3. Find $m\angle FCB$.

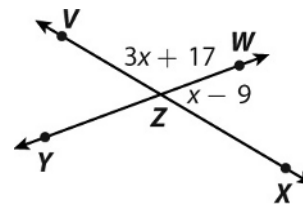
4. Find $m\angle ACG$.

Find the value of x in each figure.

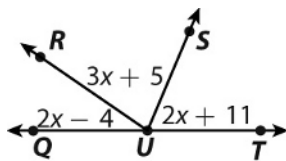
5.



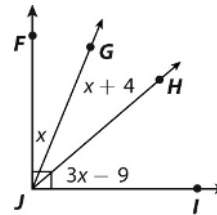
6.



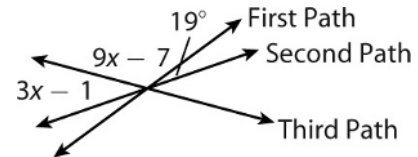
7.



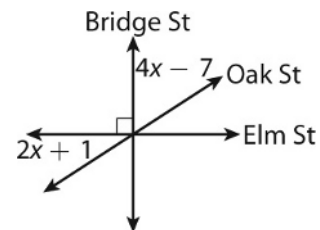
8.



9. The walking paths at a park meet each other as shown. What is the measure of the angle between the Second Path and Third Path?



10. Three streets meet each other as shown. What is the measure of the angle between Oak Street and Elm Street?

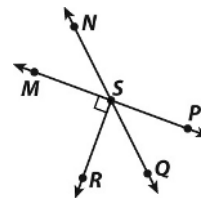


LESSON
8-4

Angle Relationships

Practice and Problem Solving: D

For Exercises 1–3, use the figure. The first one is done for you.



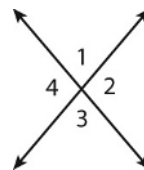
1. Name a pair of vertical angles.

$\angle PSQ$ and $\angle MSN$

2. Name a pair of complementary angles.

3. Name a pair of supplementary angles.

Use the diagram to find each angle measure. The first one is done for you.



4. If $m\angle 3 = 60^\circ$, find $m\angle 1$.

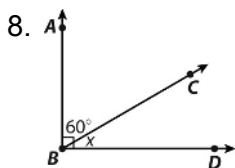
$\angle 1$ and $\angle 3$ are vertical angles,
so $m\angle 1 = 60^\circ$

5. If $m\angle 4 = 100^\circ$, find $m\angle 2$.

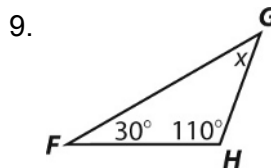
6. If $m\angle 1 = 50^\circ$, find $m\angle 2$.

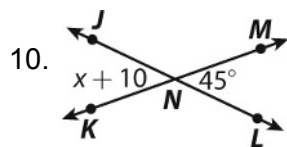
7. If $m\angle 2 = 125^\circ$, find $m\angle 3$.

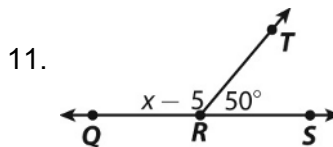
Find the value of x in each figure. The first one is done for you.



$m\angle ABC + m\angle CBD = 90$, so $x = 30$



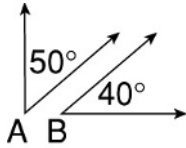
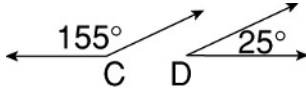
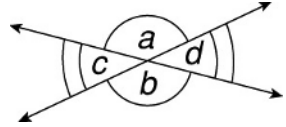




LESSON
8-4

Angle Relationships

Reteach

Complementary Angles	Supplementary Angles	Vertical Angles
		
Two angles whose measures have a sum of 90° .	Two angles whose measures have a sum of 180° .	Intersecting lines form two pairs of vertical angles.

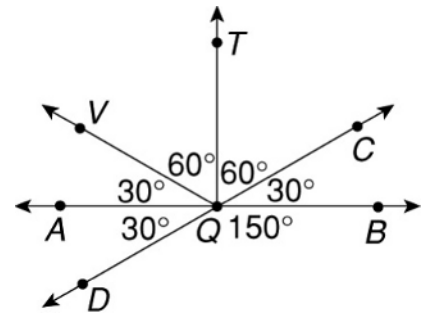
Use the diagram to complete the following.

1. Since $\angle AQC$ and $\angle DQB$ are formed by intersecting lines, \overline{AQB} and \overline{CQD} , they are:

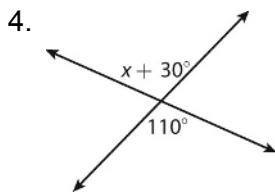
2. The sum of the measures of $\angle AQV$ and $\angle VQT$ is: _____
So, these angles are:

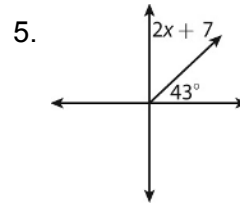
3. The sum of the measures of $\angle AQC$ and $\angle CQB$ is: _____

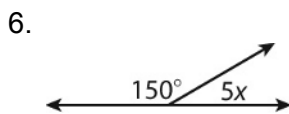
So, these angles are: _____

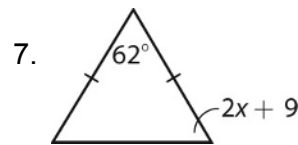


Find the value of x in each figure.









LESSON
8-4

Angle Relationships

Reading Strategies: Understanding Vocabulary

You can use definitions of angles to determine the measures of missing angles.

Pairs of angles can be classified into different types.

Complementary angles	Supplementary angles	Adjacent angles	Vertical angles	Congruent angles
The angle measures equal 90° .	The angle measures equal 180° .	The angles share a common vertex and side.	The angles are nonadjacent angles formed by two intersecting lines.	The angles have the same measure.

Find $m\angle DFE$.

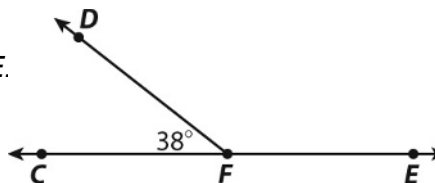
$\angle DFE$ and $\angle CFD$ are supplementary angles. So the angle measures equal 180° . You can write an equation to find $m\angle DFE$.

$m\angle DFE + m\angle CFD = 180^\circ$ Original equation

$m\angle DFE + 38^\circ = 180^\circ$ Substitute 38° for $m\angle CFD$.

$m\angle DFE + 38^\circ - 38^\circ = 180^\circ - 38^\circ$ Subtract 38° from both sides.

$m\angle DFE = 142^\circ$ Simplify.



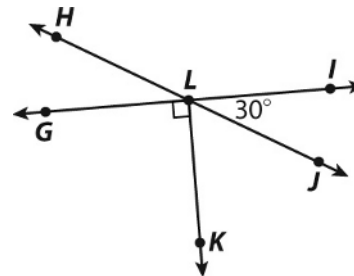
Use the diagram to find each angle measure. Show your work.

- Find $m\angle GLH$.

- Find $m\angle KLJ$.

- Find $m\angle HLI$.

- Find $m\angle ILK$.



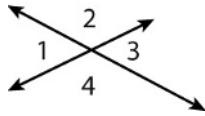
LESSON
8-4

Angle Relationships

Success for English Learners

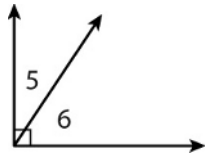
Problem 1

Vertical angles



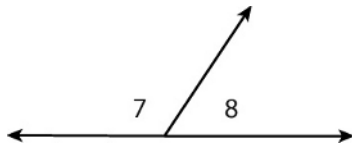
$\angle 1$ and $\angle 3$ are vertical angles.
 $\angle 2$ and $\angle 4$ are vertical angles.
 Vertical angles have the same measure.

Complementary angles.



$\angle 5$ and $\angle 6$ are complementary angles.
 Complementary angles have a sum of 90° .

Supplementary angles.



$\angle 7$ and $\angle 8$ are supplementary angles.
 Supplementary angles have a sum of 180° .

Problem 2

What is $m\angle x$?

$$\begin{aligned}
 25^\circ + 37^\circ + x &= 180^\circ \\
 62^\circ + x &= 180^\circ \\
 \underline{-62^\circ} \quad \underline{-62^\circ} & \\
 x &= 118^\circ
 \end{aligned}$$

1. What is the sum of complementary angles? Supplementary angles?

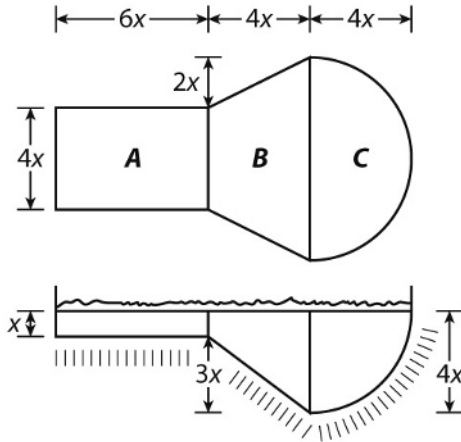
2. What is the sum of the measures of the angles in any triangle?

MODULE
8

Modeling Geometric Figures

Challenge

A city has built a combination diving and swimming pool for an international swimming competition. The diagram shows the top and side views of the pool. The dimensions are in feet.



1. What shape is the solid represented by Region A? Find its volume in cubic feet.

2. What shape is the Region B water surface? Find its area. (Hint: Its area is $\frac{1}{2}h(b_1 + b_2)$.)

3. To find the volume of Region B, start by finding the volume of the “slab” of water that is “ x ” feet deep and in the shape found in Exercise 2.

4. Next, notice that the volume of Region B below the “slab” of water in Exercise 3 is a “half” prism. What is the height of that prism? Find its volume.

5. Now, find the total volume of Region B by adding the volumes from Exercise 3 and 4.

6. What shape is the diving well, Region C? (Hint: Its volume is $\frac{4}{3}\pi r^3$.)

What part of the volume of the figure is represented by Region C?
Find that volume.

7. Find the *total* volume of the swimming pool, Regions A, B, and C.

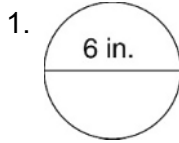
8. What value of x would make the total volume of the pool 33,000 cubic feet?

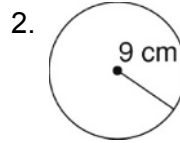
LESSON
9-1

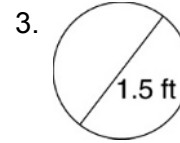
Circumference

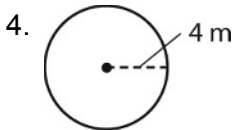
Practice and Problem Solving: A/B

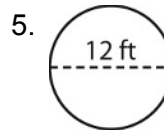
Find the circumference of each circle. Use 3.14 or $\frac{22}{7}$ for π . Round to the nearest hundredth, if necessary.

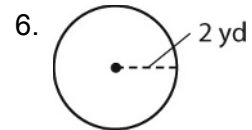


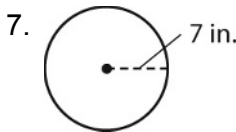


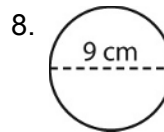


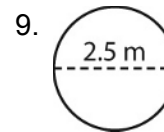












Solve.

10. A circular swimming pool is 21 feet in diameter. What is the circumference of the swimming pool? Use $\frac{22}{7}$ for π .

11. A jar lid has a diameter of 42 millimeters. What is the circumference of the lid? Use $\frac{22}{7}$ for π .

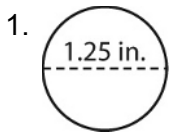
12. A frying pan has a radius of 14 centimeters. What is the circumference of the frying pan? Use $\frac{22}{7}$ for π .

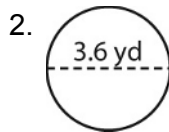
LESSON
9-1

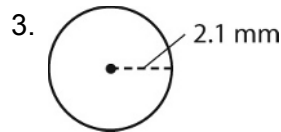
Circumference

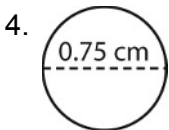
Practice and Problem Solving: C

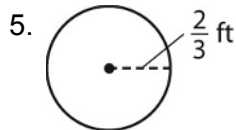
Find the circumference of each circle. Use 3.14 or $\frac{22}{7}$ for π . Round to the nearest hundredth, if necessary.

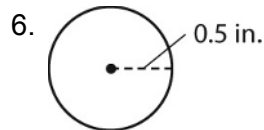












Solve each problem.

7. The circumference of a clock is 22 inches. What is the radius of the clock?

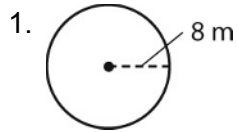
8. The circumference of a circular hot tub at a hotel is 56.5 yards. What is the diameter of the hot tub?

9. In NCAA basketball rules, the basketball can have a maximum circumference of 30 inches. What is the maximum diameter of an NCAA basketball to the nearest hundredth?

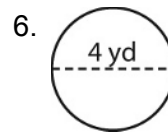
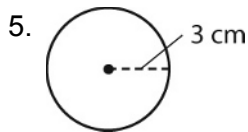
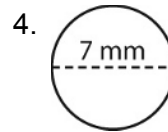
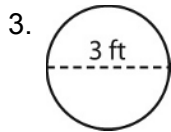
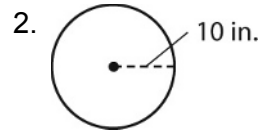
10. Melanie wants to put ribbon around the circumference of a circular section of the city park. Ribbon comes in rolls of 40 feet. The radius of the section of the park is 100 feet. How many rolls of ribbon should Melanie buy?

LESSON
9-1**Circumference****Practice and Problem Solving: D**

Find the circumference of each circle. Use 3.14 or $\frac{22}{7}$ for π . Round to the nearest tenth, if necessary. The first one is done for you.



$$C = 2\pi r \approx 2(3.14)(8) \approx 50.24; 50.2 \text{ m}$$



Solve each problem.

7. A circular patio has a diameter of 35 yards. What is the circumference of the patio? Use $\frac{22}{7}$ for π .
-

8. A paper plate has a diameter of 9 inches. What is the circumference of the plate? Use $\frac{22}{7}$ for π .
-

9. A circular light fixture has a radius of 20 centimeters. What is the circumference of the light fixture? Use $\frac{22}{7}$ for π .
-

LESSON

9-1

Circumference

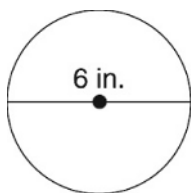
Reteach

The distance around a circle is called the **circumference**. To find the circumference of a circle, you need to know the diameter or the radius of the circle.

The ratio of the circumference of any circle to its diameter $\left(\frac{C}{d}\right)$

is always the same. This ratio is known as π (pi) and has a value of approximately 3.14.

To find the circumference C of a circle if you know the diameter d , multiply π times the diameter. $C = \pi \cdot d$, or $C \approx 3.14 \cdot d$.



$$C = \pi \cdot d$$

$$C \approx 3.14 \cdot d$$

$$C \approx 3.14 \cdot 6$$

$$C \approx 18.84$$

The circumference is about 18.8 in. to the nearest tenth.

The diameter of a circle is twice as long as the radius r , or $d = 2r$.

To find the circumference if you know the radius, replace d with $2r$ in the formula. $C = \pi \cdot d = \pi \cdot 2r$

Find the circumference given the diameter.

1. $d = 9$ cm

$$C = \pi \cdot d$$

$$C \approx 3.14 \cdot \underline{\hspace{2cm}}$$

$C \approx \underline{\hspace{2cm}}$
 The circumference is _____ cm to the nearest tenth of a centimeter.

Find the circumference given the radius.

2. $r = 13$ in.

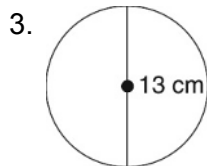
$$C = \pi \cdot 2r$$

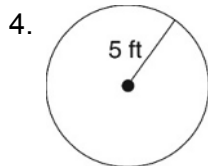
$$C \approx 3.14 \cdot (2 \cdot \underline{\hspace{2cm}})$$

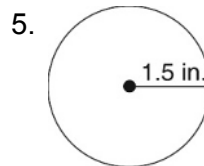
$C \approx 3.14 \cdot \underline{\hspace{2cm}}$
 $C \approx \underline{\hspace{2cm}}$
 The circumference is _____ in. to the nearest tenth of an inch.

Find the circumference of each circle to the nearest tenth.

Use 3.14 for π .







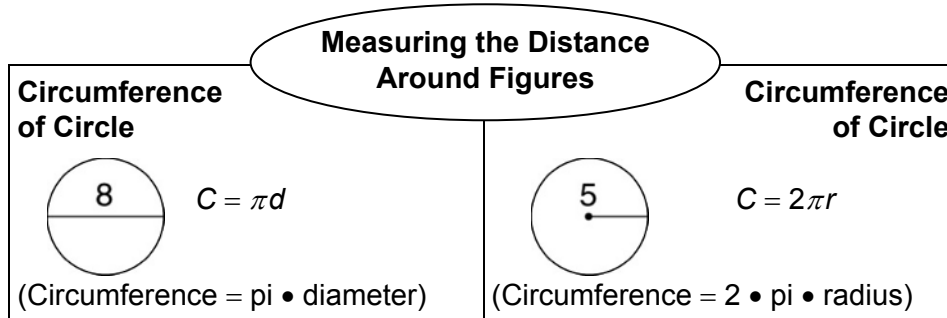
LESSON
9-1

Circumference

Reading Strategies: Using a Graphic Organizer

Perimeter is the distance around a polygon.

The chart below shows formulas for finding the circumference of circles.



Use the information in the chart above to complete each exercise.

1. If you knew the radius of a circle, what formula would you use to find its circumference?

2. If you knew the diameter of a circle, what formula would you use to find its circumference?

3. How does the length of the diameter of a circle relate to the length of the radius of that same circle?

4. What values of π can you use to approximate the circumference of a circle?

5. How does the circumference of a circle relate to the perimeter of a polygon?

LESSON

9-1

Circumference**Success for English Learners****Problem 1**

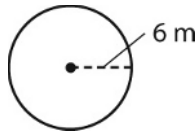
When you know the length of the radius of a circle, use the formula $C = 2\pi r$ to find its circumference.

$$C = 2\pi r$$

$$C = 2\pi(6)$$

$$C \approx 2(3.14)(6)$$

$$C \approx 37.68 \text{ m}$$

**Problem 2**

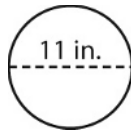
When you know the length of the diameter of a circle, use the formula $C = \pi d$ to find its circumference.

$$C = \pi d$$

$$C = \pi(11)$$

$$C \approx 3.14(11)$$

$$C \approx 34.54 \text{ in.}$$



1. What information do you need to know to use the formula $C = \pi d$?

2. A circle has a radius of 9 centimeters. What is the length of its diameter?

3. Suppose you know a circle has a diameter of 34 feet. How could you use the formula $C = 2\pi r$ to find its circumference?

4. Find the circumference of a circle with a diameter of 10 meters using both formulas. Show your work.

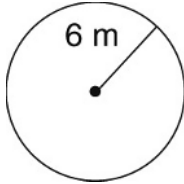
LESSON
9-2

Area of Circles

Practice and Problem Solving: A/B

Find the area of each circle to the nearest tenth. Use 3.14 for π .

1.



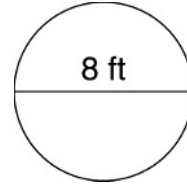
A 113 m^2

B 37.7 m^2

C 354.9 m^2

D 452.16 m^2

2.



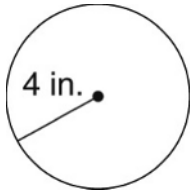
A 201 ft^2

B 50.2 ft^2

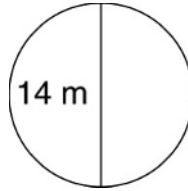
C 25.1 ft^2

D 157.8 ft^2

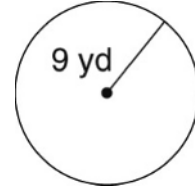
3.



4.

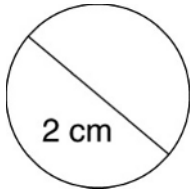


5.

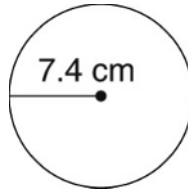


Find the area of each circle in terms of π .

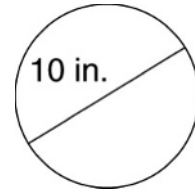
6.



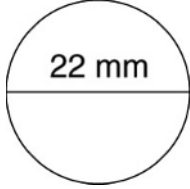
7.



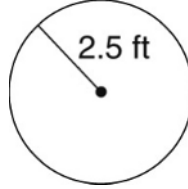
8.



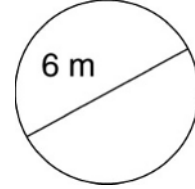
9.



10.



11.

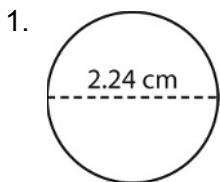


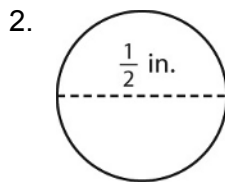
LESSON
9-2

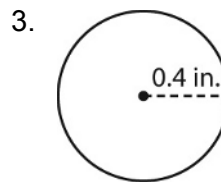
Area of Circles

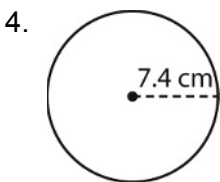
Practice and Problem Solving: C

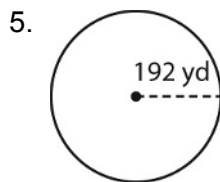
Find the area of each circle in terms of π . Then find the area to the nearest tenth using 3.14 for π .

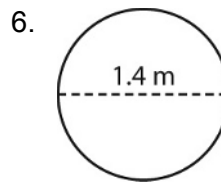












Use the formula $C^2 = 4\pi A$ to find the area A of each circle in terms of π .

7. $C = 2\pi$

8. $C = 5\pi$

9. $C = 8\pi$

Solve.

10. A vanilla cake has a diameter of 8 inches. A chocolate cake has a diameter of 10 inches. What is the difference in area between the top surfaces of the two cakes? Use 3.14 for π .

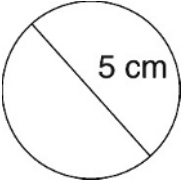
11. What is the difference in area between a circle with a diameter of 3 meters and a square with a side length of 3 meters? Use 3.14 for π .

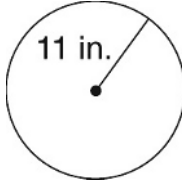
LESSON
9-2

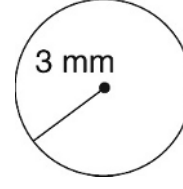
Area of Circles

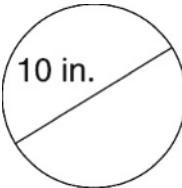
Practice and Problem Solving: D

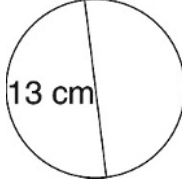
Find the area of each circle to the nearest tenth. Use 3.14 for π . The first problem is done for you.

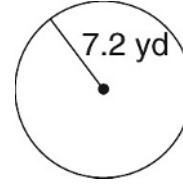
1. 
 19.6 cm²

2. 

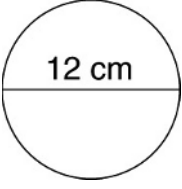
3. 


4. 

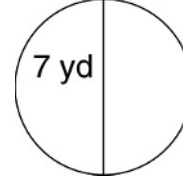
5. 

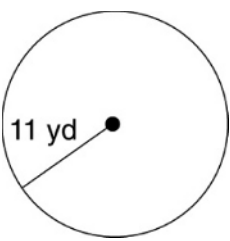
6. 

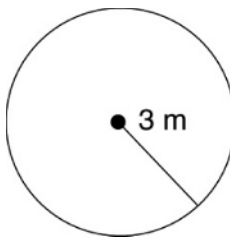
Find the area of each circle in terms of π . The first problem is done for you.

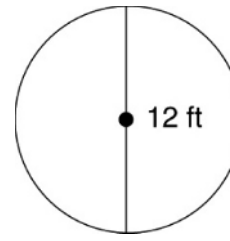
7. 
 36π cm²

8. 

9. 

10. 

11. 

12. 

LESSON

9-2

Area of Circles

Reteach

The area of a circle is found by using the formula $A = \pi r^2$. To find the area, first determine the radius. Square the radius and multiply the result by π . This gives you the exact area of the circle.

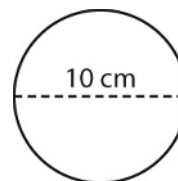
Example:

Find the area of the circle in terms of π .

The diameter is 10 cm. The radius is half the diameter, or 5 cm. Area is always given in square units.

$$5^2 = 25$$

$$A = 25\pi \text{ cm}^2$$



Find the area of each circle in terms of π .

1. A vinyl album with a diameter of 16 inches. _____
2. A compact disc with a diameter of 120 mm. _____

Sometimes it is more useful to use an estimate of π to find your answer. Use 3.14 as an estimate for π .

Example:

Find the area of the circle. Use 3.14 for π and round your answer to the nearest tenth.

The radius is 2.8 cm.

Area is always given in square units.

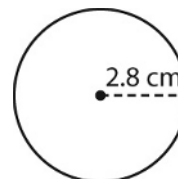
$$2.8^2 = 7.84$$

$$A = 7.84\pi \text{ cm}^2$$

$$A = 7.84 \times 3.14 \text{ cm}^2$$

$$A = 24.6176 \text{ cm}^2$$

Rounded to the nearest tenth, the area is 24.6 cm^2 .



Find the area of each circle. Use 3.14 for π and round your answer to the nearest tenth.

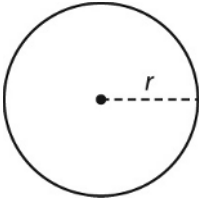
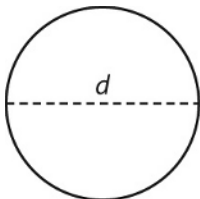
3. A pie with a radius of 4.25 inches. _____
4. A horse ring with a radius of 10 yards. _____

5. A round pond with a diameter of 24 m. _____
6. A biscuit with a diameter of 9.2 cm. _____

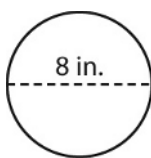
LESSON
9-2

Area of Circles

Reading Strategies: Make Connections

<p>Radius</p>  $A = \pi r^2$ $\pi \approx 3.14 \text{ or } \frac{22}{7}$	<p>Diameter</p>  $A = \pi r^2$ $r = \frac{d}{2}$ $A = \pi \left(\frac{d}{2} \right)^2$
--	--

Find the area of each circle in terms of π . Then find the estimated area using 3.14 for π .



Think: Do I know the diameter or the radius?
The radius goes across half the circle. The diameter goes across the whole circle.

The diameter. I can find the radius by dividing the diameter by 2.

$$A = \pi r^2$$

$$r = \frac{d}{2}$$

$$A = \pi \left(\frac{8}{2} \right)^2 = \pi \cdot 4^2 = 16\pi$$

*In terms of π , the area is $16\pi \text{ in.}^2$
To find the estimated area, use 3.14 for π .*

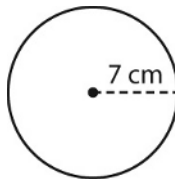
$$A = 16\pi$$

$$= 16 \cdot 3.14$$

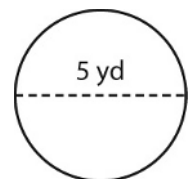
$$= 50.24 \text{ in}^2$$

Find the area of each circle in terms of π . Then find the estimated area using 3.14 for π .

1. _____



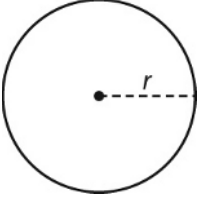
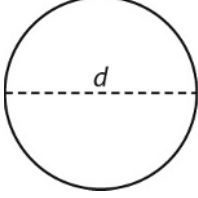
2. _____



LESSON
9-2

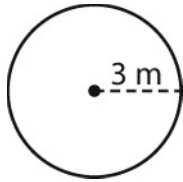
Area of Circles

Success for English Learners

 $A = \pi r^2$ $\pi \approx 3.14 \text{ or } \frac{22}{7}$	 $A = \pi r^2$ $r = \frac{d}{2}$ $A = \pi \left(\frac{d}{2}\right)^2$
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Problem 1

A. The radius is given.



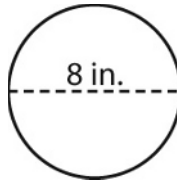
$$A = \pi r^2 \quad r = 3$$

$$A = \pi \cdot 3^2 = 9\pi$$

$$\approx 9 \cdot 3.14$$

$$\approx 28.3 \text{ m}^2$$

B. The diameter is given.



$$A = \pi r^2$$

$$r = \frac{d}{2} = \frac{8}{2} = 4$$

$$A = \pi \cdot 4^2 = 16\pi$$

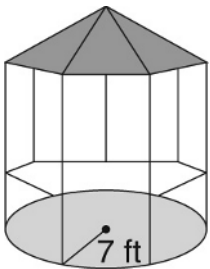
$$\approx 16 \cdot 3.14$$

$$\approx 50.2 \text{ in.}^2$$

Problem 2

Find the area in terms of π .

A. The radius is given.



$$A = \pi r^2 \quad r = 7$$

$$A = \pi \cdot 7^2 = 49\pi$$

$$A = 49\pi \text{ ft}^2$$

B. The diameter is given.



$d = 9 \text{ in.}$

$$A = \pi r^2$$

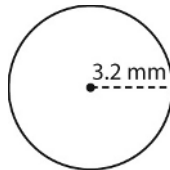
$$r = \frac{d}{2} = \frac{9}{2} = 4.5$$

$$A = \pi \cdot 4.5^2 = 20.25\pi$$

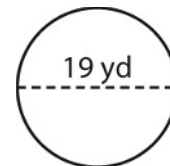
$$A = 20.25\pi \text{ in.}^2$$

Find the area in terms of π . Then use 3.14 for π and find the area to the nearest tenth.

1. _____



2. _____

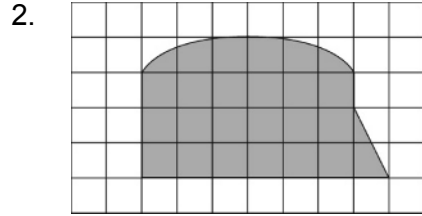
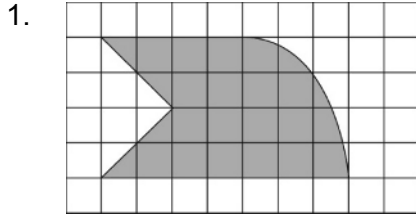


LESSON
9-3

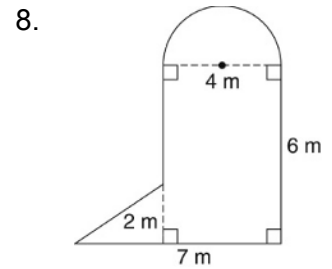
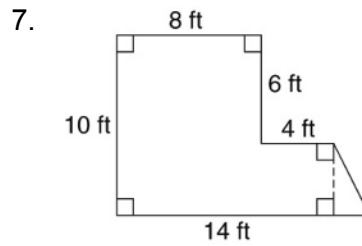
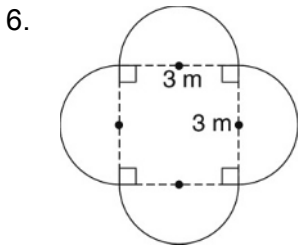
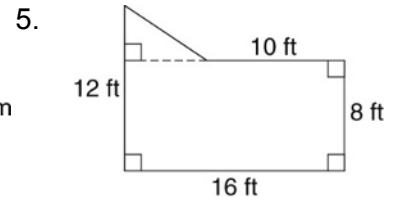
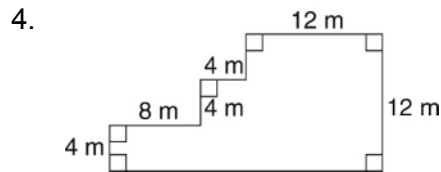
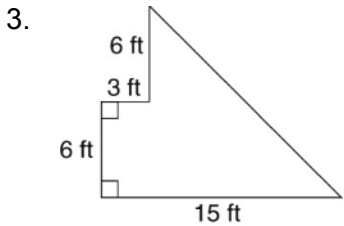
Area of Composite Figures

Practice and Problem Solving: A/B

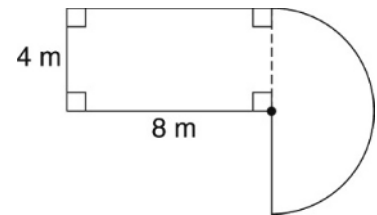
Estimate the area of each figure. Each square represents 1 square foot.



Find the area of each figure. Use 3.14 for π .



9. Marci is going to use tile to cover her terrace. How much tile does she need?

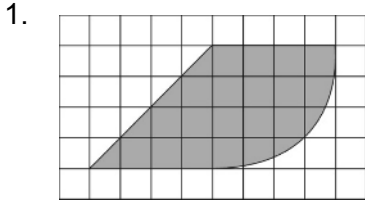


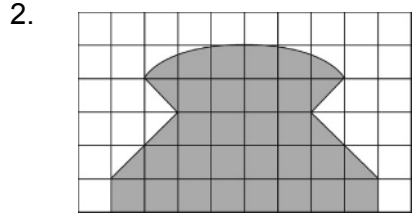
LESSON
9-3

Area of Composite Figures

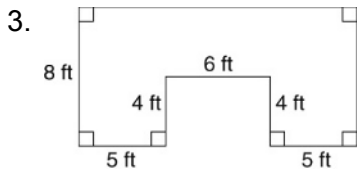
Practice and Problem Solving: C

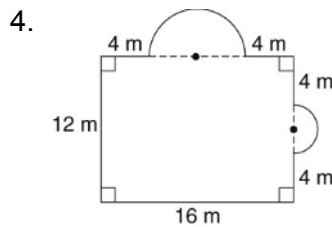
Estimate the area of each figure. Each square represents 1 square foot.

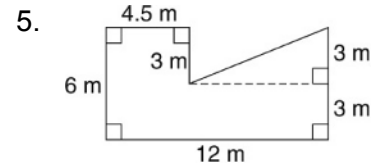




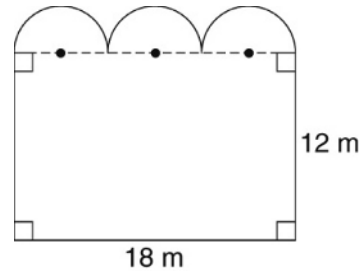
Find the area of each figure. Use 3.14 for π .



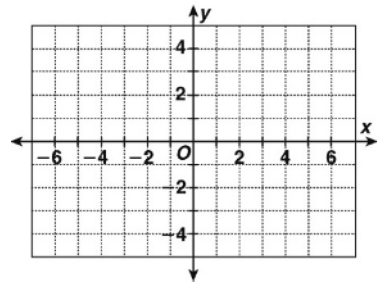




6. The figure shows the dimensions of a room in which receptions are held. The room is being carpeted. The three semi-circular parts of the room are congruent. How much carpet is needed?



7. A polygon has vertices at $F(-5, 2)$, $G(-3, 2)$, $H(-3, 4)$, $J(1, 4)$, $K(1, 1)$, $L(4, 1)$, $M(4, -2)$, $N(6, -2)$, $P(6, -3)$, and $Q(-5, -3)$. Graph the figure on the coordinate plane. Then find the area and perimeter of the figure.

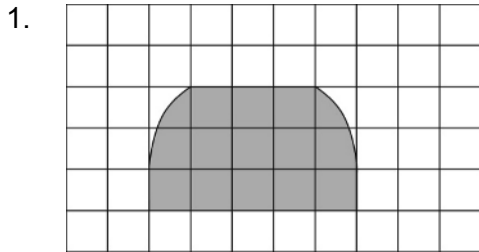


LESSON
9-3

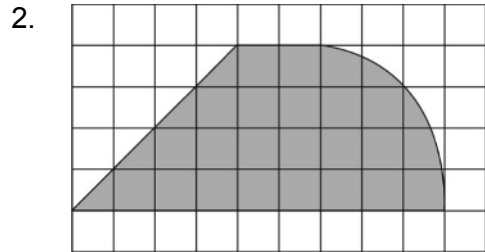
Area of Composite Figures

Practice and Problem Solving: D

Estimate the area of each figure. Each square represents 1 square foot. Choose the letter for the best answer. The first one is done for you.

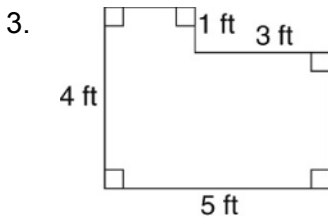


- A 10 ft² © 14 ft²
B 11 ft² D 15 ft²

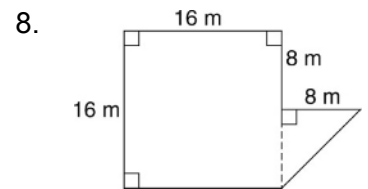
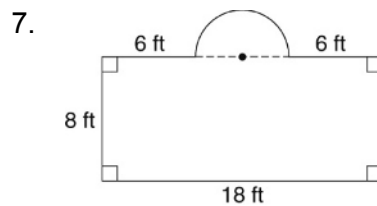
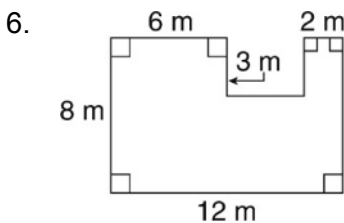
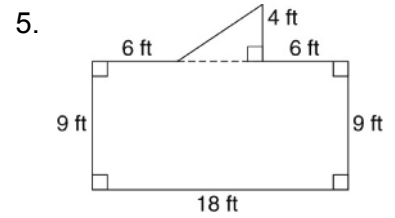
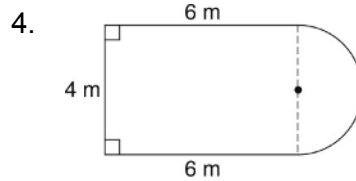


- A 24 ft² C 32 ft²
B 26 ft² D 36 ft²

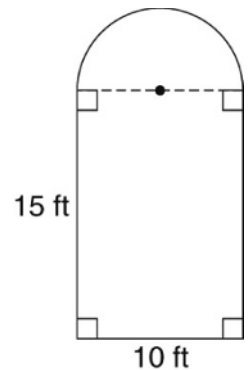
Find the area of each figure. Use 3.14 for π . The first one is done for you.



17 ft²



9. The figure shows the dimensions of a room. How much carpet is needed to cover its floor?



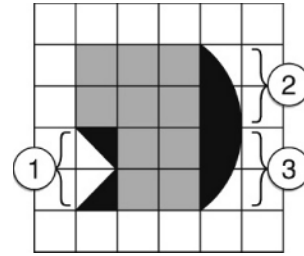
LESSON
9-3

Area of Composite Figures

Reteach

When an irregular figure is on graph paper, you can estimate its area by counting whole squares and parts of squares. Follow these steps.

- Count the number of whole squares. There are 10 whole squares.
- Combine parts of squares to make whole squares or half-squares.



Section 1 = 1 square

Section 2 $\approx 1\frac{1}{2}$ squares

Section 3 $\approx 1\frac{1}{2}$ squares

- Add the whole and partial squares

$$10 + 1 + 1\frac{1}{2} + 1\frac{1}{2} = 14$$

The area is about 14 square units.

Estimate the area of the figure.

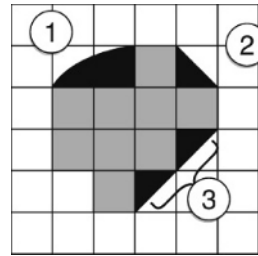
1. There are _____ whole squares in the figure.

Section 1 \approx _____ square(s)

Section 2 = _____ square(s)

Section 3 = _____ square(s)

$A = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$ square units

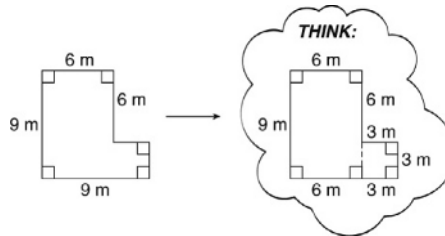


You can break a composite figure into shapes that you know. Then use those shapes to find the area.

A (rectangle) = $9 \times 6 = 54 \text{ m}^2$

A (square) = $3 \cdot 3 = 9 \text{ m}^2$

A (composite figure) = $54 + 9 = 63 \text{ m}^2$

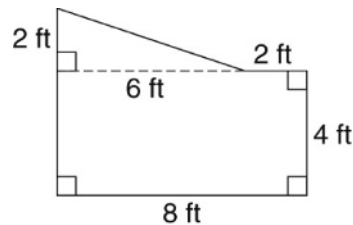


Find the area of the figure.

2. A (rectangle) = _____ ft^2

A (triangle) = _____ ft^2

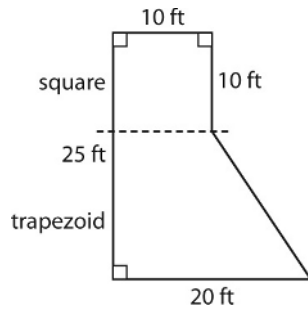
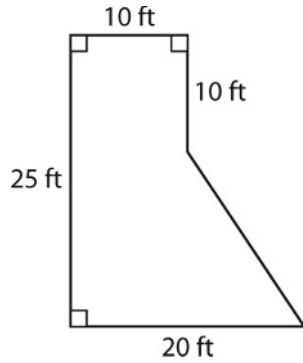
A (composite figure) = _____ + _____ = _____ ft^2



LESSON
9-3

Area of Composite Figures

Reading Strategies: Make Connections



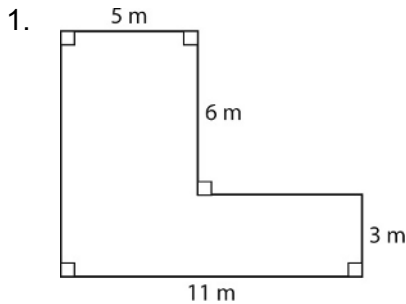
Shape	Area Formula
Triangle	$A = \frac{1}{2}bh$
Square	$A = s^2$
Rectangle	$A = lw$
Parallelogram	$A = bh$
Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$
Circle	$A = \pi r^2$

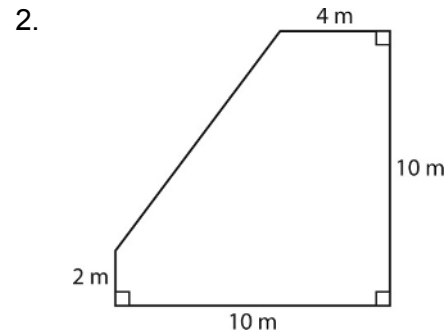
square = $s^2 = 10^2 = 100 \text{ ft}^2$

trapezoid = $\frac{1}{2} \cdot 25(10 + 20) = 12.5(30) = 375 \text{ ft}^2$

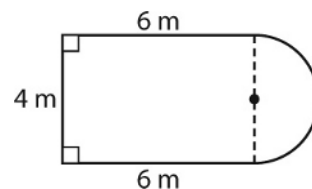
square + trapezoid = $100 + 375 = 475 \text{ ft}^2$

Find the area of each figure. Use 3.14 for π .





3. The figure shows the dimensions of a room. How much carpet is needed to cover the floor of the room?



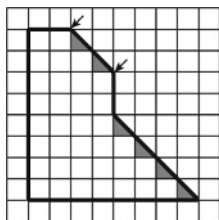
LESSON
9-3

Area of Composite Figures

Success for English Learners

Problem 1

Find the area.



Think!
Count the two kinds of squares and add.

Step 1: Count the number of whole squares: 35

Step 2: Count the number of half squares: 6

Think!
Make the half-filled squares full.

Step 3: $6 \div 2 = 3$

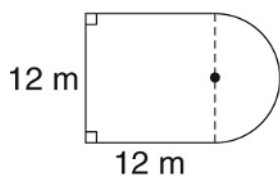
Think!
Add to find the area.

Step 4: $35 + 3 = 38$

So, the area is 38 square units.

Problem 2

Find the area.



Think!
Find familiar figures. Then use the formulas for areas.

Figure 2 is a semicircle

Area of semicircle:

$$A = \frac{1}{2}(\pi r^2)$$

Use 3.14 for π .

$$r = 6 \text{ so } r^2 = 36$$

$$A \approx \frac{1}{2}(3.14 \cdot 36)$$

Substitute.

$$A \approx \frac{1}{2}(113.04)$$

Multiply.

$$A \approx 56.52 \text{ m}^2$$

Multiply by $\frac{1}{2}$.

Think!
The radius is $\frac{1}{2}(12)$.

Figure 1 = square

Area of square: $A = s^2$

$$s = 12$$

$$s^2 = 12^2 = 144$$

$$A = 144 \text{ m}^2$$

$$\text{Total area} \approx 144 + 56.52 \approx A \approx 200.52 \text{ m}^2$$

1. How can you find the area of composite figures?

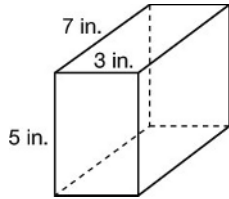
LESSON
9-4

Solving Surface Area Problems

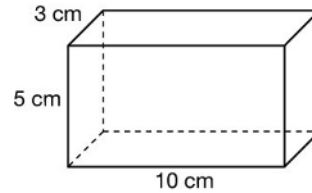
Practice and Problem Solving: A/B

Find the surface area of each solid figure.

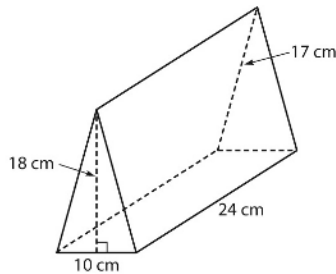
1. _____



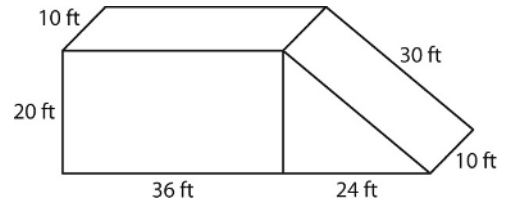
2. _____



3. _____

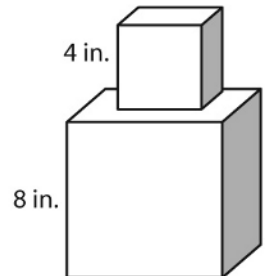


4. _____



Use the situation below to complete Exercises 5–6.

Cydney built a display stand out of two cubes. The larger cube is 8 inches on each side. The smaller cube is 4 inches on each side. She painted the display stand after she put the two cubes together. She did NOT paint the bottom of the display stand. What was the total area she painted?



5. Explain your plan for solving the problem.

6. Solve the problem.

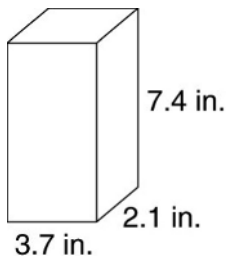
LESSON
9-4

Solving Surface Area Problems

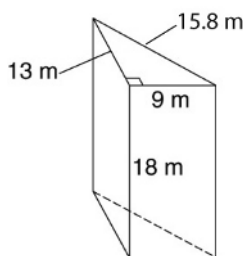
Practice and Problem Solving: C

Find the surface area of each solid figure. Round each answer to the nearest tenth.

1. _____

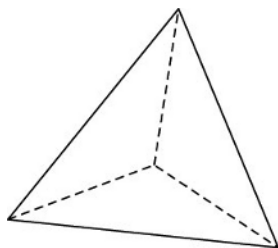


2. _____

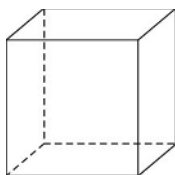


Three students constructed three-dimensional figures of cardboard.

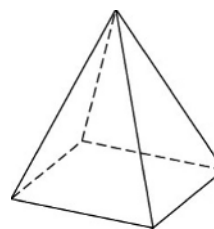
A.



B.



C.



3. Read each description in Exercises 4–6. Before doing any calculations, guess which figure has the greatest surface area and which has the least surface area.

4. Josh made his figure from 6 congruent squares. The edge of each square was 8 inches. Which figure did Josh construct? What is the surface area of his figure?

5. Kayla used 4 isosceles and one square for her figure. Each edge of the square was 10 inches. The two congruent sides of the triangles are 13 inches long. The height of the triangles is 12 inches. Which figure did Kayla construct? What is the surface area of her figure?

6. Angelica used four congruent equilateral triangles. Each side of the triangles was 14 inches. The height of each triangle was 12.1 inches. Which figure did she construct? What is the surface area of her figure?

7. Check your guesses from Exercise 3. Were you correct? _____

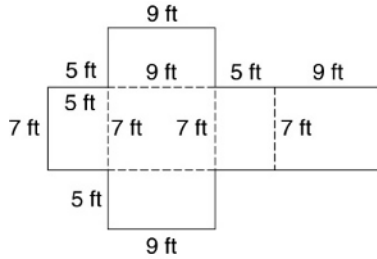
LESSON
9-4

Solving Surface Area Problems

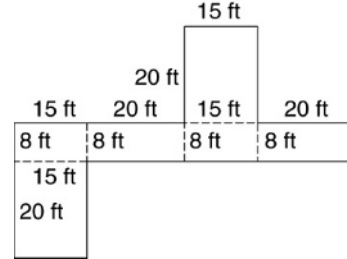
Practice and Problem Solving: D

Find the surface area of each solid figure. The first one is done for you.

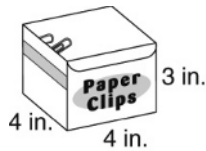
1. 286 ft²



2. _____



3. _____



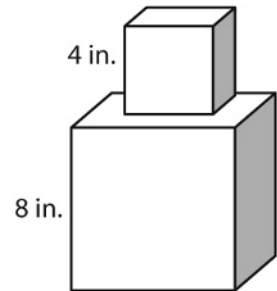
4. _____



Use the situation below to answer 5–7.

Cydney built a display stand out of two cubes. The larger cube is 8 inches on each side. The smaller cube is 4 inches on each side. She painted the entire outside of each cube before she put the cubes together.

5. What was the surface area she painted for the smaller cube? _____
6. What was the surface area she painted for the larger cube? _____
7. What was the total area that she painted on both cubes? _____



LESSON
9-4

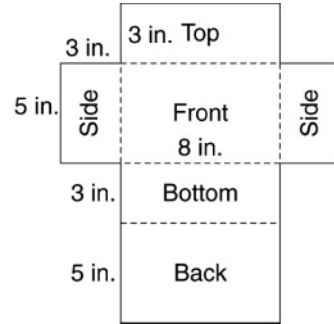
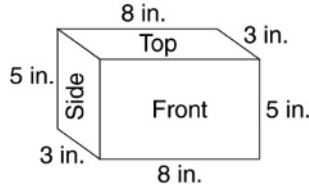
Solving Surface Area Problems

Reteach

The surface area of a three-dimensional figure is the combined areas of the faces.

You can find the surface area of a prism by drawing a **net** of the flattened figure.

Notice that the top and bottom have the same shape and size. Both sides have the same shape and size. The front and the back have the same shape and size.



Remember: $A = lw$

Since you are finding area, the answer will be in square units.

Find the surface area of the prism formed by the net above.

1. Find the area of the front face: $A = \underline{\quad} \cdot \underline{\quad} = \underline{\hspace{2cm}} \text{ in}^2$.

The area of the front and back faces is $2 \cdot \underline{\quad} = \underline{\hspace{2cm}} \text{ in}^2$.

2. Find the area of the side face: $A = \underline{\quad} \cdot \underline{\quad} = \underline{\hspace{2cm}} \text{ in}^2$.

The area of the 2 side faces is $2 \cdot \underline{\quad} = \underline{\hspace{2cm}} \text{ in}^2$.

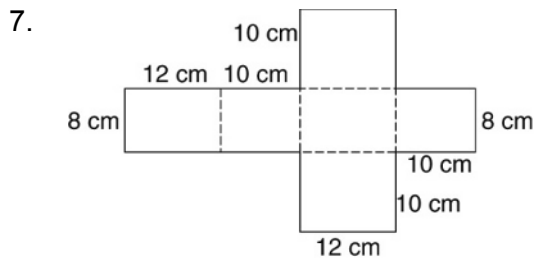
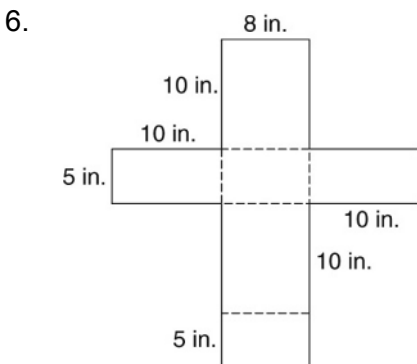
3. Find the area of the top face: $A = \underline{\quad} \cdot \underline{\quad} = \underline{\hspace{2cm}} \text{ in}^2$.

The area of the top and bottom faces is $2 \cdot \underline{\quad} = \underline{\hspace{2cm}} \text{ in}^2$.

4. Combine the areas of the faces: $\underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\hspace{2cm}} \text{ in}^2$.

5. The surface area of the prism is $\underline{\hspace{2cm}} \text{ in}^2$.

Find the surface area of the prism formed by each net.



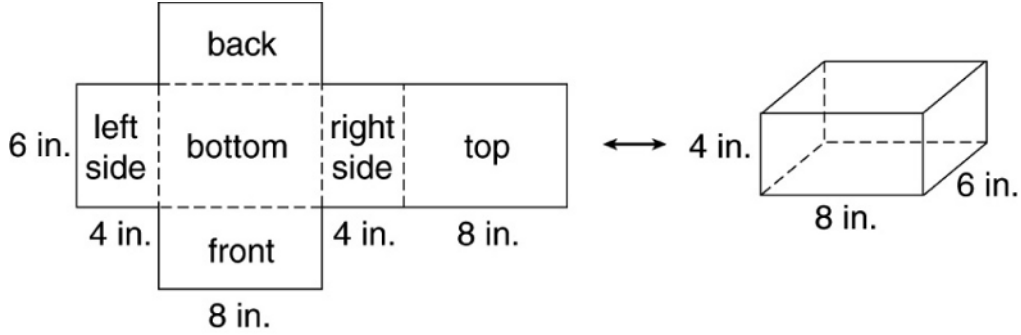
LESSON
9-4

Solving Surface Area Problems

Reading Strategies: Analyze Information

The **surface area** of a three-dimensional figure is the total area of all its surfaces.

If you analyze the net of a rectangular prism, you notice there are six faces. Each face pairs up with another, congruent face:



To find the surface area of a rectangular prism, find the sum of the areas of the six faces. Since you are finding area, the answer will be in square units.

Remember: $A = lw$.

Use the congruent pairs of faces to simplify the computation.

Left side and right side: $2 \times (6 \times 4) = 48$

Back and front: $2 \times (8 \times 4) = 64$

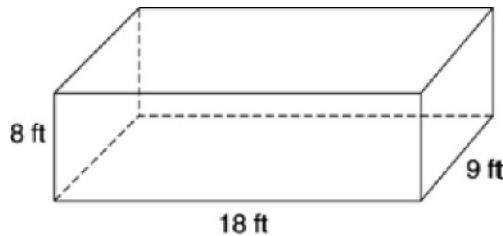
Top and bottom: $2 \times (6 \times 8) = 96$

Sum of areas: $48 + 64 + 96 = 208$

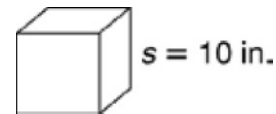
Total surface area: 208 square inches

Find the surface area of each rectangular prism.

1.



2.

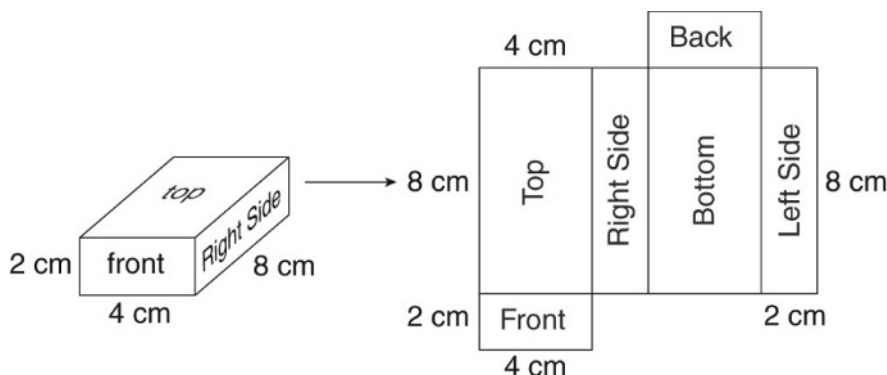


LESSON
9-4

Solving Surface Area Problems

Success for English Learners

If you unfold a three-dimensional figure and lay it flat, you have made a net.



A **net** is a two-dimensional shape that lets you picture all the surfaces of a three-dimensional figure. A net helps you see how much surface a three-dimensional figure covers.

The **surface area** of a three-dimensional figure is the total area of all its surfaces. Surface area is measured in square units. Remember: $A = lw$

Complete.

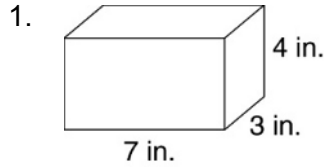
1. What is the area of the top? _____
2. What is the area of the bottom? _____
3. What is the area of the front? _____
4. What is the area of the back? _____
5. What is the area of the left side? _____
6. What is the area of the right side? _____
7. What is the total surface area of the figure? _____
8. What do you notice about pairs of surfaces of a rectangular prism that have the same areas?

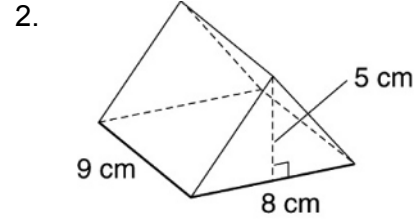
LESSON
9-5

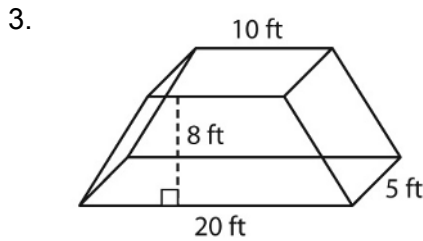
Solving Volume Problems

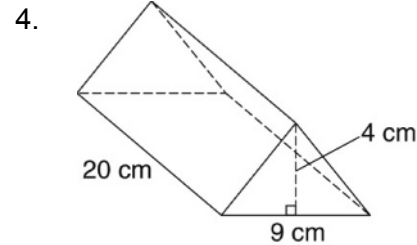
Practice and Problem Solving: A/B

Find the volume of each figure.







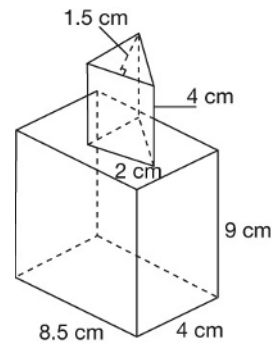


Using cheese, Theo made the display shown at right. Use the figure to complete Exercises 5–7.

5. How many cubic centimeters of cheese are in the completed display?

6. Each kilogram of the cheese Theo used takes up a volume of about 20 cubic centimeters. What is the approximate mass of Theo's display?

7. Theo's friend made a display with dimensions that were each half as long as those Theo used. What is the approximate mass of Theo's friend's display?

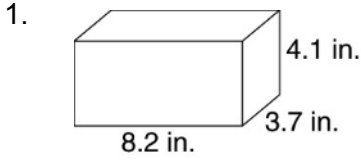


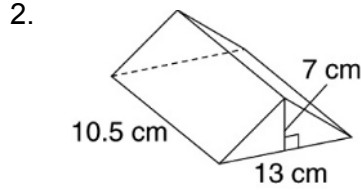
LESSON
9-5

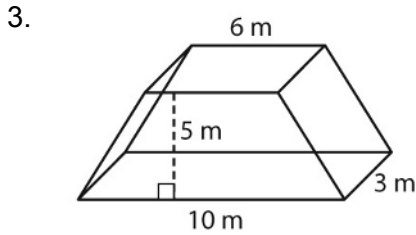
Solving Volume Problems

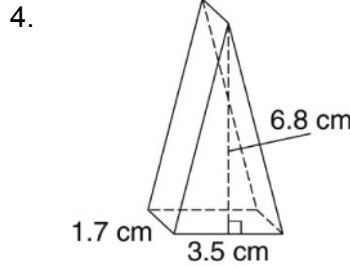
Practice and Problem Solving: C

Find the volume of each figure.

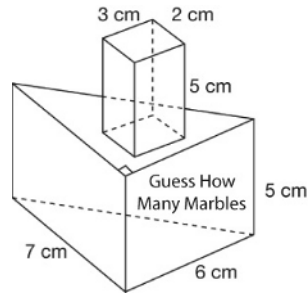








For the school carnival, Pietro built a clear plastic container to be used in a game called **Guess How Many Marbles**. The container is to be filled with marbles that are 12 millimeters in diameter.



5. Find the volume of the entire container.

6. Marsha says that one marble has a volume of 904.3 cm^2 , so the container can't hold any marbles. What error did Marsha make?

7. Can you find how many marbles fit in the container by dividing the volume of the container by the volume of 1 marble? Explain your reasoning.

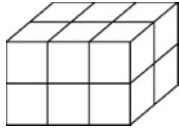
LESSON
9-5

Solving Volume Problems

Practice and Problem Solving: D

Tell how many cubes are in each figure. The first one is done for you.

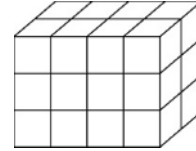
1.



Think: There are 3×2 cubes in each layer. There are 2 layers. So, there are $3 \times 2 \times 2$ cubes.

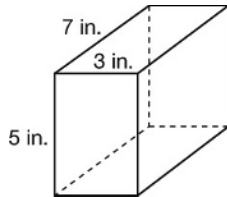
12 cubes

2.



Find the volume of each figure. The first one is done for you.

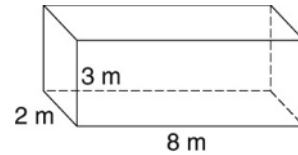
3.



Think: $V = lwh$
 $V = 7 \times 3 \times 5 = 105$

105 in^3

4.

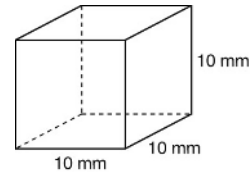


For a school project, students have to build a pyramid of cubes.

5. Each cube will be like the one shown at the right. What are the dimensions of the cube?

length: _____ mm height: _____ mm

width: _____ mm

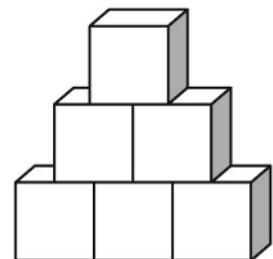


6. What is the volume of the cube at the right?

The completed pyramid will look like the figure shown at the right.

7. How many cubes are in the pyramid?

8. What is the volume of the entire pyramid?



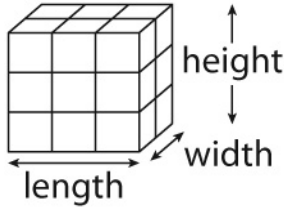
LESSON
9-5

Solving Volume Problems

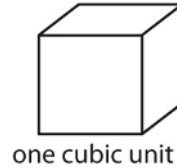
Reteach

The **volume** of a solid figure is the number of cubic units inside the figure.

A prism is a solid figure that has length, width, and height.



Each small cube represents one cubic unit.

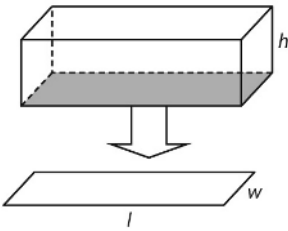


Volume is measured in cubic units, such as in^3 , cm^3 , ft^3 , and m^3 .

The volume of a solid figure is the product of the area of the base (B) and the height (h).

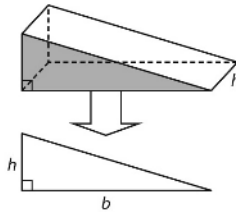
$V = Bh$

Rectangular Prism



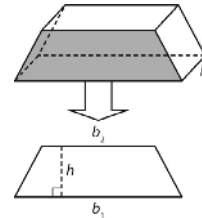
The base is a rectangle. To find the area of the base, use $B = lw$.

Triangular Prism



The base is a triangle. To find the area of the base, use $B = \frac{1}{2}bh$.

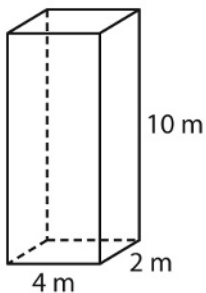
Trapezoidal Prism



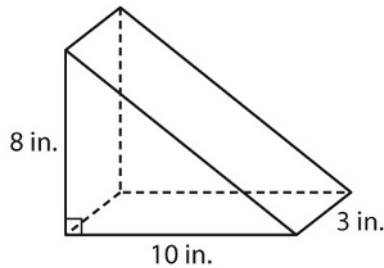
The base is a trapezoid. To find the area of the base, use $B = \frac{1}{2}(b_1 + b_2)h$.

Find the volume of each figure.

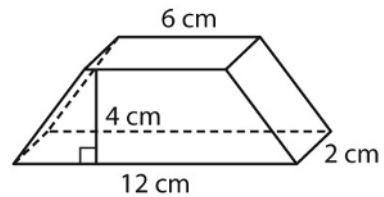
1.



2.



3.

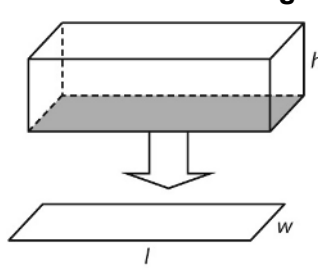
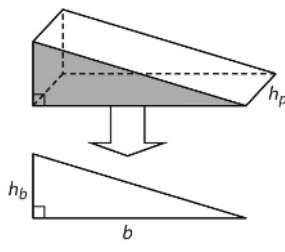
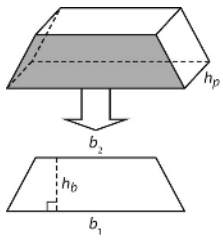


LESSON
9-5

Solving Volume Problems

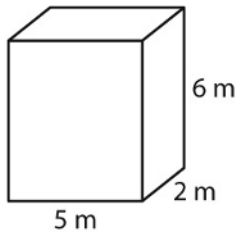
Reading Strategies: Use a Graphic Organizer

You can use a graphic organizer to help when you are solving volume problems.

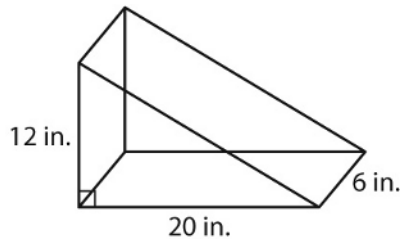
<p>Volume of all Prisms</p> $V = Bh$ <p> ↑ ↑ ↑ volume area of height of of a the base of the prism of prism prism </p>	<p>Volume of a Rectangular Prism</p>  <p>The base is a rectangle. To find the area of the base, use $B = lw$.</p> $V = Bh = lwh$
<p>Volume of a Triangular Prism</p>  <p>The base is a triangle. To find the area of the base, use $B = \frac{1}{2}bh$.</p> $V = Bh = \frac{1}{2}bh_bh_p$	<p>Volume of a Trapezoidal Prism</p>  <p>The base is a trapezoid. To find the area of the base, use $B = \frac{1}{2}(b_1 + b_2)h$.</p> $V = Bh = \frac{1}{2}(b_1 + b_2)h_bh_p$

Find the volume of each figure.

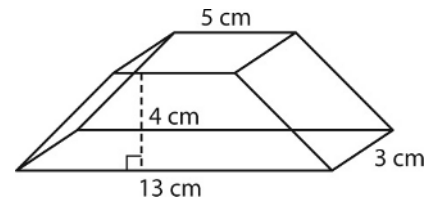
1.



2.



3.



LESSON
9-5

Solving Volume Problems

Success for English Learners

The volume of a prism is equal to the area of the base of the prism times the height of the prism.

Problem 1

Some prisms have more than one measure that is labeled “height.”

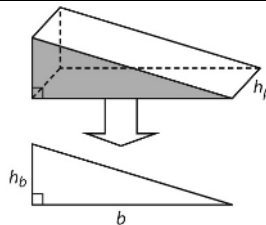
Volume of a Triangular Prism

In a triangular prism, there is the height of the base triangle and the height of the prism.

Sometimes subscripts (small letters to the right of a letter) can help you know which “height” to use.

h_b is the height of the base.

h_p is the height of the prism.



The base is a triangle. To find the area of the

base, use $B = \frac{1}{2}bh_b$.

$$V = Bh = \frac{1}{2}bh_bh_p$$

Problem 2

Some prisms have more than one measure that is labeled “base.”

Volume of a Trapezoidal Prism

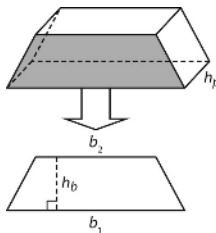
In a trapezoidal prism, the area of the base is the average of the lengths of the two bases times the height.

The subscripts tell you to use both bases (b_1 and b_2) when using this formula.

The trapezoidal prism also has two “heights.”

h_b is the height of the base.

h_p is the height of the prism.



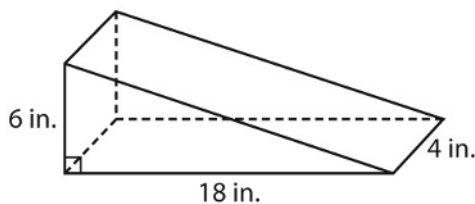
The base is a trapezoid. To find the area of the

base, use $B = \frac{1}{2}(b_1 + b_2)h_b$.

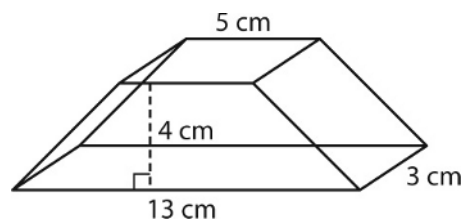
$$V = Bh = \frac{1}{2}(b_1 + b_2)h_bh_p$$

Find the volume of each figure.

1.



2.



MODULE
9
Circumference, Area, and Volume
Challenge

The Schultz family is planning a garden for part of their back yard. The width of the garden will be 20 feet. Each family member has different requirements. The requirements are shown in the table below.

Name	Requirement
Mr. Schultz	The maximum length of fencing around the garden is 100 feet.
Mrs. Schultz	The area of the garden must be greater than 400 square feet.
Angelica	One-half of the garden will be flowers. At most, there will be 350 square feet for flowers.
Robert	At least 15% percent of the garden will be used to grow carrots and lettuce. At the least, there will be 45 square feet for carrots and lettuce.

Write and solve an inequality that represents the possible values for the widths of the garden for each family member. Then write an inequality that would satisfy everyone.

- Mr. Schultz: _____
- Mrs. Schultz: _____
- Angelica: _____
- Robert: _____
- Dimensions for garden that satisfies all 4 people:
width (w): _____ length (l): _____

The family also wants to put a circular fish pond in the backyard. The pond is to be at least 2 feet deep and have an area of at least $78\frac{4}{7}$ ft² and no greater than 154 ft². A 2-foot wide walkway will surround the pond. The family creates a pond that meets the requirements.

Use $\pi = \frac{22}{7}$. Use that information to answer each question.

- What are the possible dimensions of the radius (r) of the pond? _____
- What are the possible lengths of the circumference (C_p) of the pond? _____
- What are the possible lengths of the circumference (C_w) of the outside of the walkway? _____
- What are the possible values of the volume (V) of the pond? _____

UNIT 4: Geometry

MODULE 8 Modeling Geometric Figures

LESSON 8-1

Practice and Problem Solving: A/B

- 15 ft; 6 ft; 90 ft²
- 16 m; 12 m; 192 m²
- The scale drawing is 10 units by 8 units.
- a. 1 ft = 125 m
b. 84 sheets of plywood tall
- a. 40 bottle caps tall
b. approximately 3 popsicle sticks tall

Practice and Problem Solving: C

- 25.5 ft; 23.8 ft; 606.9 ft²
- Because the scale is 8 mm: 1 cm, and because 1 cm is longer than 8 mm, the actual object will be larger.
- a. 42 cm by 126 cm
b. 5,292 cm²
c. approximately 1.386 ft by 4.158 ft
d. approximately 5.763 ft²
- 64 in.
- 35.2 ft

Practice and Problem Solving: D

1.

Blueprint length (in.)	5	10	15	20	25	30
Actual Length (ft)	8	16	24	32	40	48

- 48 ft
- 2.5 in.

2.

Blueprint length (in.)	2	4	6	8	10	12
Actual Length (ft)	1	2	3	4	5	6

- 6 ft
- 16 in.

- 24 ft; 12 ft; 288 ft²
- 10 units by 8 units

Reteach

- 3 in.; 24 in.; $\frac{1}{8}$
- 4 cm; 20 cm; $\frac{1}{5}$
- 84 in.
- 75 mi

Reading Strategies

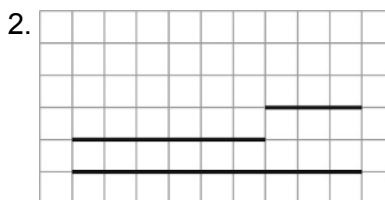
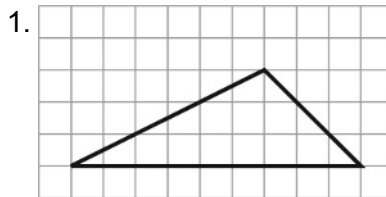
- 3 cm
- Sample answer: $\frac{1}{10} = \frac{3}{x}$
- 5 cm
- Sample answer: $\frac{1}{10} = \frac{5}{x}$

Success for English Learners

- Sample answer: The car would not be in proportion.
- Sample answer: If the photo does not have the same proportions as the painting, the face will be stretched tall or stretched wide.

LESSON 8-2

Practice and Problem Solving: A/B



No triangle can be formed because the sum of the measures of the two shorter sides has the same measure as the longest side.

- Yes, because the sum of the measures of the two shorter sides is greater than the measure of the longest side,

$$\text{e.g., } \frac{1}{3} + \frac{1}{4} > \frac{1}{2}.$$

- No, because the sum of the measures of the two shorter sides is less than the measure of the longest side, e.g., $0.02 + 0.01 < 0.205$.
- Unique; since the sum of the angles is less than 180° and a side is included.
- Many, since the sum of the measures of the angles is less than 180° but no side is included.

Practice and Problem Solving: C

- They are angles ACB and ADB , formed by Earth's radii and the tangent lines running to the planet.
- Both are Earth's radii.
- AC is much less than BC .
- AB and BC are approximately equal.
- $AB > BC$
- Isosceles triangle, since AB and BC are approximately equal.
- The astronomer knows that ACB is a right angle and the angle CAB could be measured. This is enough information to compute AB using similar triangles or trigonometry.

Practice and Problem Solving: D

- 3 and 4 units; less than 7 units, but greater than 1 unit; Diagrams will vary.
- 3 and 7 units; less than 10 units, but greater than 4 units; Diagrams will vary.
- 101° ; 79°
- 129° ; 51°

Reteach

- Yes; if x is the length of each side, then $x + x > x$ or $2x > x$, so the condition for a triangle to be formed is met.
- No. The sum of the measures of the three angles is greater than 180° .

Reading Strategies

- Diagrams may vary, but students should realize that the two 4-foot boards add up

to 8 feet, which is less than the 10-foot board, so no triangle can be formed with the boards.

- Diagrams and calculations may vary, but students should first find the hypotenuse of the right triangle formed by the 5 and 6-inch sides, which is $\sqrt{61}$ inches. Then, they should find the length of the hypotenuse formed by the 25-inch side and $\sqrt{61}$ inches, which is $\sqrt{686}$ inches, or about 26 inches. A 30-inch bat would not fit in the box.

Success for English Learners

- The compass could be used to make two arcs of radii equal in length to the shorter segments from each end of the longer segment. The point of intersection of the arcs would be where the shorter sides of the triangle intersect.
- Yes, the sum of the measures of the angles given is 90° , so the third angle has to be 90 degrees for the sum of the three angle measures to be 180° .

LESSON 8-3

Practice and Problem Solving: A/B

- cross section; The circle is a plane figure intersecting a three-dimensional curved surface. The figure formed is a curved line on the surface of the cone.
- intersection; The edge of a square is a straight line and the base of the pyramid is a plane figure. A straight line is formed.
- cross section; A square is formed.
- cross section; The circle is a plane figure. A polygon results that is similar to the polygon that forms the base.
- trapezoid
- triangle
- circle
- ellipse or oval

Practice and Problem Solving: C

- It is a square. The length of each of its sides is the same as the length of the side of the square.
- An equilateral triangle; Since each of the segments from the vertex of the cube to the midpoint of the side is equal and the

angles at the vertex are 90° , the third sides of each triangle are equal and form the cross section.

3. A: circle; B and C: ellipses or ovals; D: a plane of length, h , the cylinder's height, and width, d , the cylinder's diameter
4. Area A < Area B < Area C < Area D

Practice and Problem Solving: D

1. a triangle that is similar to the base
2. a rectangle or a square
3. a trapezoid
4. a circle
5. Drawings will vary, but the cross section should be a regular octagon that is congruent to the bases of the prism.
6. Drawings will vary, but the cross section should be a regular pentagon that is similar to the base of the pyramid.

Reteach

1. Drawings will vary. Sample answers: a triangular cross section formed by a plane that is perpendicular to the base of the pyramid and including its apex point; a rectangular cross section formed by a plane that is parallel to the base of the pyramid
2. Drawings will vary, Sample answers: a triangular cross section formed by a plane that is parallel to the prism's bases and congruent to them; a rectangular cross section formed by a plane that is perpendicular to the bases and having a length that is equal to the height of the prism

Reading Strategies

1. Diagrams will vary but should show a rectangular cross section that is parallel to the base and similar to it.
2. rectangle
3. Diagrams will vary but should show a pentagonal cross section that is congruent to the bases.
4. parallel to the bases
5. congruent to bases

6. Diagrams will vary but should show a circular cross section of radius less than the radius of the sphere.
7. circle
8. similar to a circle that is the circumference of the sphere but smaller than that circle
9. Diagrams will vary but should show a plane passing through the cone's vertex, its lateral surface in two lines, and bisecting its base.
10. isosceles triangle
11. The two sides of the triangle that are equal length are the same length as the slant height of the cone. The third, shorter side is equal to the diameter of the cone's base.

Success for English Learners

1. It is a trapezoid; the edge of the cross section in the base is longer than and parallel to the edge of the cross section in the face of the pyramid.
2. Both cross sections are parallel to the bases. Each cross section is similar to the figure's base.

LESSON 8-4

Practice and Problem Solving: A/B

1. $\angle AEB$ and $\angle DEF$
2. $\angle AEB$ and $\angle BEC$
3. Sample answer: $\angle AEF$ and $\angle DEF$
4. 120°
5. 13°
6. 70°
7. 115°
8. 28
9. 18
10. 22
11. 15

Practice and Problem Solving: C

1. 66°
2. 125°
3. 114°
4. 156°
5. 39

6. 43
7. 24
8. 19
9. 41.25°
10. 33°

Practice and Problem Solving: D

1. $\angle MSN$ and $\angle PSQ$
2. $\angle PSQ$ and $\angle QSR$
3. Sample answer: $\angle MSN$ and $\angle NSP$
4. 60°
5. 100°
6. 130°
7. 55°
8. 30
9. 40
10. 35
11. 135

Reteach

1. vertical angles;
2. 90° ; complementary angles
3. 180° ; supplementary angles
4. 80
5. 20
6. 6
7. 25

Reading Strategies

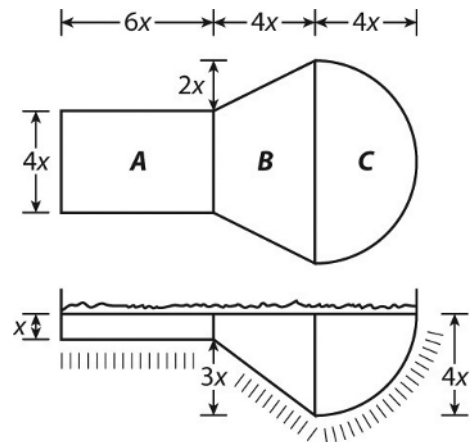
1. 30°
2. 60°
3. 150°
4. 90°

Success for English Learners

1. 90° ; 180°
2. 180°

Module 8 Challenge

1. A rectangular solid; $V_A = 4x(6x)x = 24x^3$
2. A trapezoid; $A_B = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}4x(4x + 8x) = 24x^2$
3. $V_{B \text{ part } 1} = A_B(x) = (24x^2)x = 24x^3$
4. $V_{B \text{ part } 2} = \frac{1}{2}A_B(3x) = \frac{1}{2}(24x^2)(3x) = 36x^3$
5. $V_{B \text{ total}} = 24x^3 + 36x^3 = 60x^3$
6. A sphere; one fourth of a sphere;
 $V_C = \frac{1}{4} \left(\frac{4}{3} \pi (4x)^3 \right) = \frac{64}{3} \pi x^3$
7. $V_{\text{total}} = V_A + V_{B \text{ total}} + V_C = 24x^3 + 60x^3 + \frac{64}{3} \pi x^3 = 4x^3 \left(21 + \frac{16}{3} \pi \right)$ or approx. $151x^3$.
8. Divide 33,000 by 151 to get about 218. Take the cube root; x is about 6 feet.



MODULE 9 Circumference, Area, and Volume

LESSON 9-1

Practice and Problem Solving: A/B

- 18.84 in.
- 56.52 cm
- 4.71 ft
- 25.12 m
- 37.68 ft
- 12.56 yd
- 43.96 in.
- 26.26 cm
- 7.85 m
- 66 ft
- 132 mm
- 88 cm

Practice and Problem Solving: C

- 3.93 in.
- 11.30 yd
- 13.19 mm
- 2.36 cm
- 4.19 ft
- 3.14 in.
- 3.5 in.
- 18 yd
- 9.55 in.
- 16

Practice and Problem Solving: D

- 50.2 m
- 62.8 in.
- 9.4 ft
- 22.0 mm
- 18.8 cm

- 12.6 yd
- 110 yd
- 28.3 in.
- 125.7 cm

Reteach

- 9; 28.26; 28.3
- 13; 26; 81.64; 81.6
- 40.8 cm
- 31.4 ft
- 9.4 in.

Reading Strategies

- $C = 2\pi r$
- $C = \pi d$
- It is twice as long.
- Sample answer: 3.14 or $\frac{22}{7}$
- The circumference of a circle is the distance around a circle. It is given in units. The perimeter of a polygon is the distance around a polygon. It is given in units.

Success for English Learners

- the length of the diameter.
- 18 cm
- Take half of the diameter, 17 ft, and substitute that value into the formula for r .
- $d = 10$ so $r = 5$

$$C = 2\pi r$$

$$= 2 \cdot 3.14 \cdot 5$$

$$= 31.4$$

$$C = \pi d$$

$$= 3.14 \cdot 10$$

$$= 31.4$$

LESSON 9-2

Practice and Problem Solving: A/B

1. A
2. B
3. 50.2 in.^2
4. 153.9 m^2
5. 254.3 yd^2
6. $\pi \text{ cm}^2$
7. $54.76\pi \text{ cm}^2$
8. $25\pi \text{ in.}^2$
9. $121\pi \text{ mm}^2$
10. $6.25\pi \text{ ft}^2$
11. $9\pi \text{ m}^2$

Practice and Problem Solving: C

1. $1.2544\pi \text{ cm}^2$; 3.9 cm^2
2. $0.0625\pi \text{ in.}^2$; 0.2 in^2
3. $0.16\pi \text{ in.}^2$; 0.5 in^2
4. $54.76\pi \text{ cm}^2$; 171.9 cm^2
5. $36,864\pi \text{ yd}^2$; $115,753 \text{ yd}^2$
6. $0.49\pi \text{ m}^2$; 1.5 m^2
7. $A = \pi$
8. $A = 6.25\pi$
9. $A = 16\pi$
10. The area of the 10-inch chocolate cake is 28.26 in^2 larger than the area of the vanilla cake.
11. The square's area is 1.935 m^2 larger than the circle's area.

Practice and Problem Solving: D

1. 19.6 cm^2
2. 379.9 in.^2
3. 28.3 mm^2
4. 78.5 in^2
5. 132.7 cm^2
6. 162.8 yd^2

7. $36\pi \text{ cm}^2$
8. $90.25\pi \text{ in}^2$
9. $12.25\pi \text{ yd}^2$
10. $121\pi \text{ yd}^2$
11. $9\pi \text{ m}^2$
12. $36\pi \text{ ft}^2$

Reteach

1. $64\pi \text{ in}^2$
2. $3600\pi \text{ m}^2$
3. 56.7 in.^2
4. 314 yd^2
5. 452.2 m^2
6. 66.4 cm^2

Reading Strategies

1. $49\pi \text{ cm}^2$; 153.86 cm^2
2. $6.25\pi \text{ yd}^2$; 19.625 yd^2

Success for English Learners

1. $10.24\pi \text{ mm}^2$; 32.2 mm^2
2. $90.25\pi \text{ yd}^2$; 283.4 yd^2

LESSON 9-3

Practice and Problem Solving: A/B

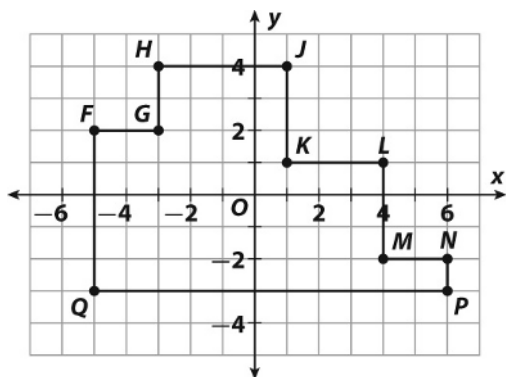
Answers may vary for Exercises 1 and 2.

1. 21 ft^2
2. 24 ft^2
3. 90 ft^2
4. 208 m^2
5. 140 ft^2
6. 23.13 m^2
7. 100 ft^2
8. 33.28 m^2
9. 57.12 m^2

Practice and Problem Solving: C

Answers may vary for Exercises 1 and 2.

- 22 ft²
- 30 ft²
- 104 ft²
- 223.4m²
- 60.75 m²
- 258.39 m²
- $A = 52 \text{ units}^2$; $P = 36 \text{ units}$



Practice and Problem Solving: D

- C
- B
- 17 ft²
- 30.28 m²
- 174 ft²
- 84 m²
- 158.13 ft²
- 288 m²
- 189.25 ft²

Reteach

- 9, $1\frac{1}{2}$, $\frac{1}{2}$, 1, 9, $1\frac{1}{2}$, $\frac{1}{2}$, 1, 12
- 32, 6, 32, 6, 38

Reading Strategies

- 63 m²
- 76 m²
- 30.28 m²

Success for English Learners

- Separate the figures into simpler figures whose areas you can find.

LESSON 9-4

Practice and Problem Solving: A/B

- 142 in²
- 190 cm²
- 1,236 cm²
- 3,380 ft²
- Possible answer: I would find the total surface area of each cube and then subtract the area of the sides that are not painted, including the square underneath the small cube.
- 384 in²

Practice and Problem Solving: C

- 101.4 in²
- 797.4 m²
- Check students' guesses.
- B; 384 in²
- C; 340 in²
- A; 338.8 in²
- Discuss students' guesses and whether they were correct or not.

Practice and Problem Solving: D

- 286 ft²
- 1,160 ft²
- 80 in²
- 124 in²
- 96 in²
- 384 in²
- 480 in²

Reteach

- $5 \bullet 8 = 40 \text{ in}^2$; $2 \bullet 40 = 80 \text{ in}^2$
- $5 \bullet 3 = 15 \text{ in}^2$; $2 \bullet 15 = 30 \text{ in}^2$
- $3 \bullet 8 = 24 \text{ in}^2$; $2 \bullet 24 = 48 \text{ in}^2$
- $80 + 30 + 48 = 158 \text{ in}^2$
- 158 in^2
- 340 in^2
- 592 cm^2

Reading Strategies

- 756 square feet
- 600 square inches

Success for English Learners

- 32 cm^2
- 32 cm^2
- 8 cm^2
- 8 cm^2
- 16 cm^2
- 16 cm^2
- 112 cm^2
- Sample answer: There are 3 pairs of surfaces with the same areas: the top and bottom, the left side and right side, the front and back.

LESSON 9-5

Practice and Problem Solving: A/B

- 84 in^3
- 180 cm^3
- 600 ft^3
- 360 cm^3
- 312 cm^3
- 15.6 kg
- 1.95 kg

Practice and Problem Solving: C

- 124.4 in^3
- 477.8 cm^3
- 120 m^3
- 20.2 cm^3
- 135 cm^3
- Marsha got the units confused. The volume of one marble is $7,234.5 \text{ mm}^3$. Marsha needs to convert that volume to cm^3 , which is about 7.2 cm^3 .
- Answers will vary. Sample answer: If you divide the volume of the container by the volume of 1 marble, you can find the number of marbles that will fit inside the container. However, the volume of all the marbles will not equal the volume of the container, because the marbles are round and there will be empty space.

Practice and Problem Solving: D

- 12 cubes
- 24 cubes
- 105 in^3
- 48 m^3
- length: 10 mm; width: 10 mm; height: 10 mm
- $1,000 \text{ mm}^3$
- 6 cubes
- $6,000 \text{ mm}^3$

Reteach

- 80 m^3
- 120 in^3
- 72 cm^3

Reading Strategies

- 60 m^3
- 720 in^3
- 108 cm^3

Success for English Learners

- 216 in^3
- 108 cm^3

MODULE 9 Challenge

1. $2(20 + x) \leq 100$; $x \leq 30$
2. $20x > 400$; $x > 20$
3. $0.5(20x) \leq 350$; $x \leq 35$
4. $0.15(20x) \geq 45$; $x \geq 15$
5. $w = 20$ feet and $30 \text{ feet} \geq l > 20$ feet
6. $5 \text{ ft} \leq r \leq 7 \text{ ft}$
7. $31\frac{3}{7} \text{ ft} \leq C_p \leq 44 \text{ ft}$
8. $44 \text{ ft} \leq C_w \leq 56\frac{1}{7} \text{ ft}$
9. $157\frac{1}{7} \text{ ft}^3 \leq V \leq 308 \text{ ft}^3$